LEARNING TO USE ARITHMETIC

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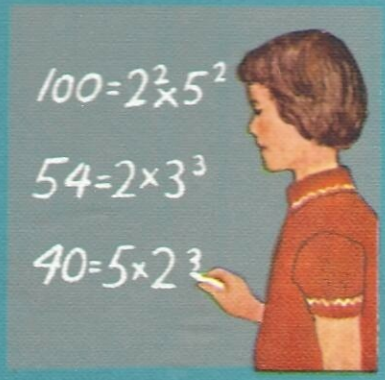

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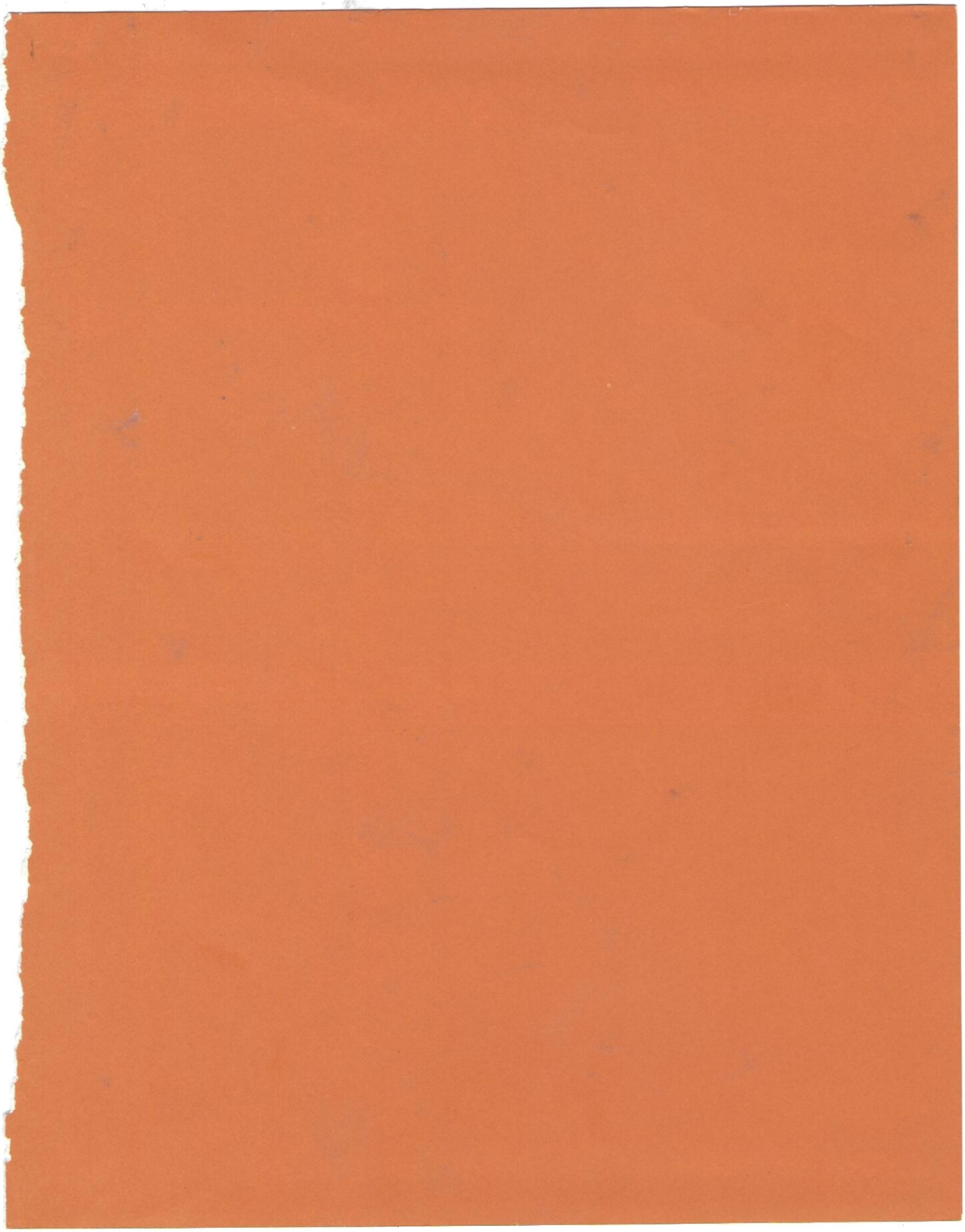
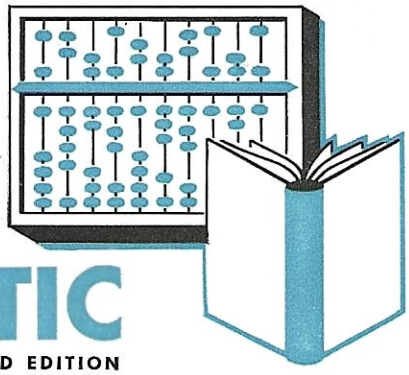


LEARNING TO USE ARITHMETIC

REVISED EDITION



BEGINNERS BOOK

BOOK 1

BOOK 2

BOOK 3

BOOK 4

BOOK 5

BOOK 6

BOOK 7

BOOK 8

AUTHORS

HOWARD F. FEHR
Teachers College, Columbia University, New York

AGNES G. GUNDERSON
Formerly University of Wyoming, Laramie

GEORGE E. HOLLISTER
University of Wyoming, Laramie

JO McKEEBY PHILLIPS
Lecturer and Consultant in Mathematics

JOSEPH H. RANDALL
Formerly Public Schools, Newton, Massachusetts

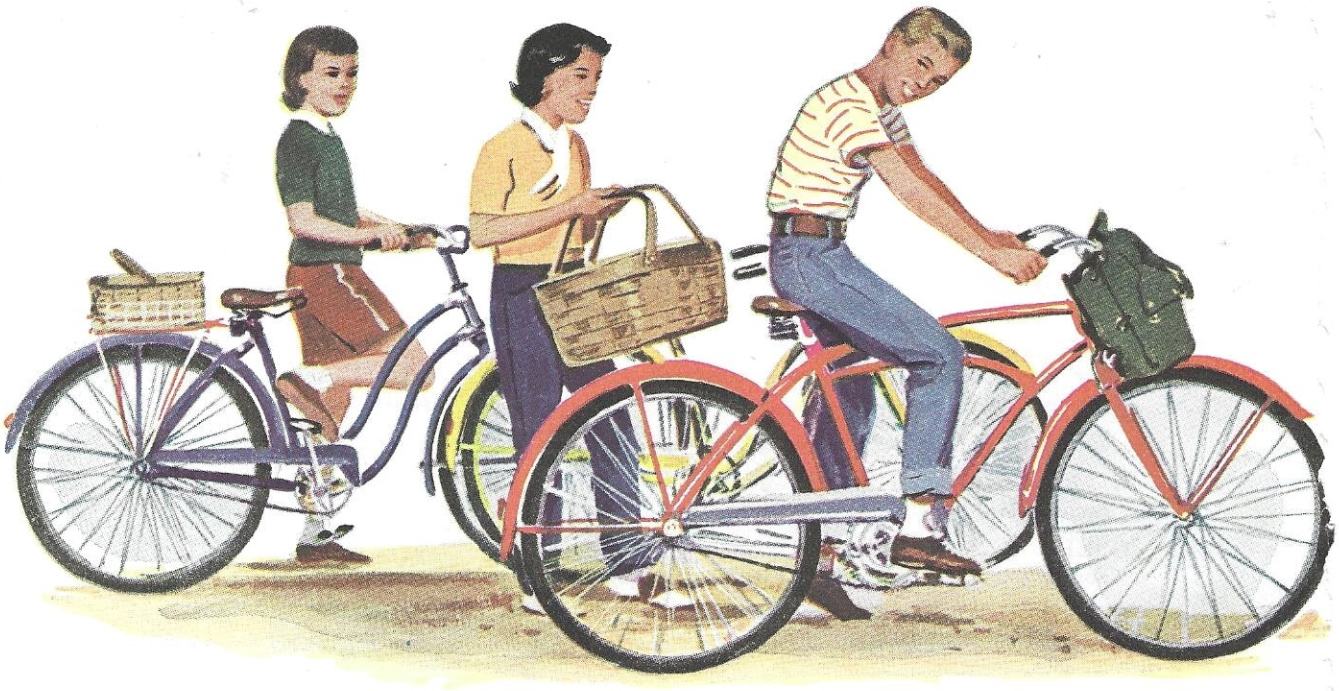
JOSEPH J. URBANCEK
Chicago Teachers College

F. LYNWOOD WREN
San Fernando Valley State College, Northridge, California

J. WAYNE WRIGHTSTONE
Bureau of Educational Research, New York City

LEARNING TO USE

BOOK 6



Illustrations by HARVEY KIDDER

RICHARD C. BARTLETT, JR. HAROLD CUE RAY GRANAI

ARITHMETIC

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BY J. WAYNE WRIGHTSTONE

in collaboration with

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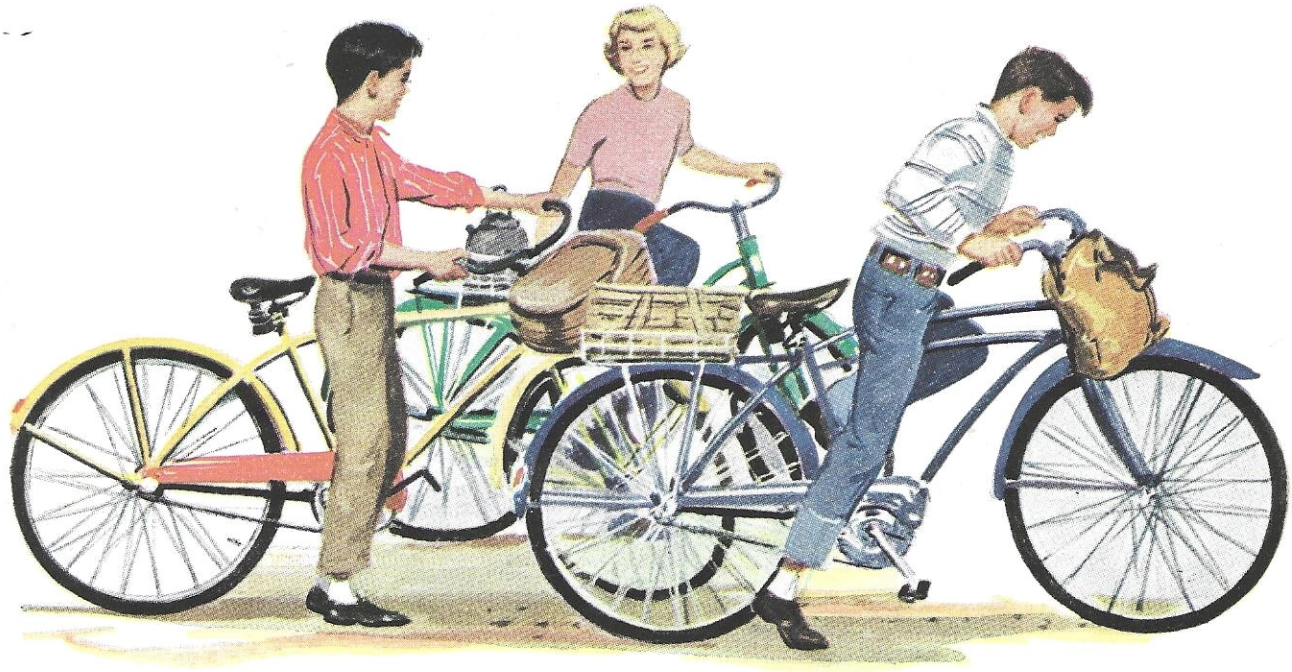
F. LYNWOOD WREN



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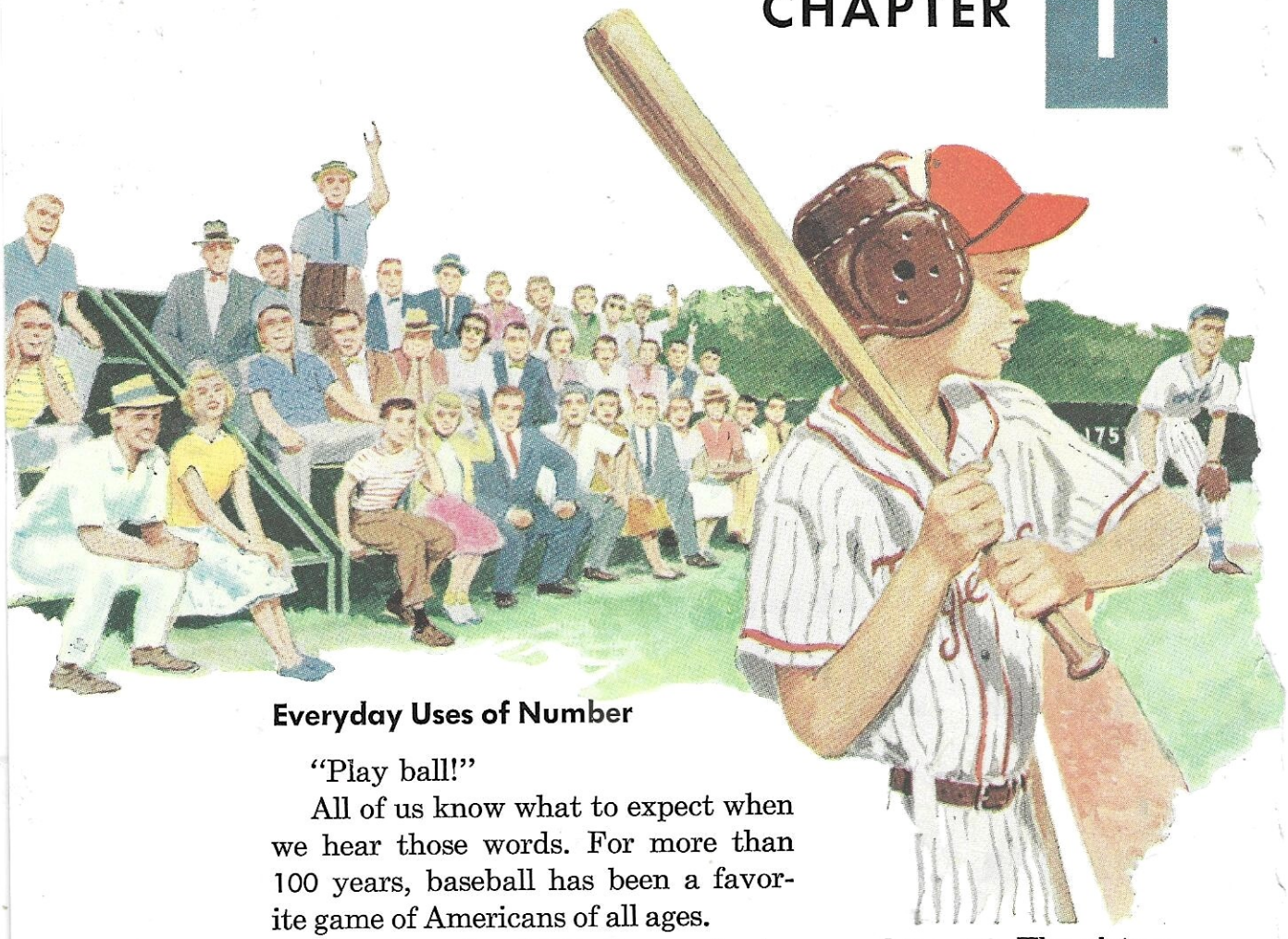
CHAPTER 1



Everyday Uses of Number

"Play ball!"

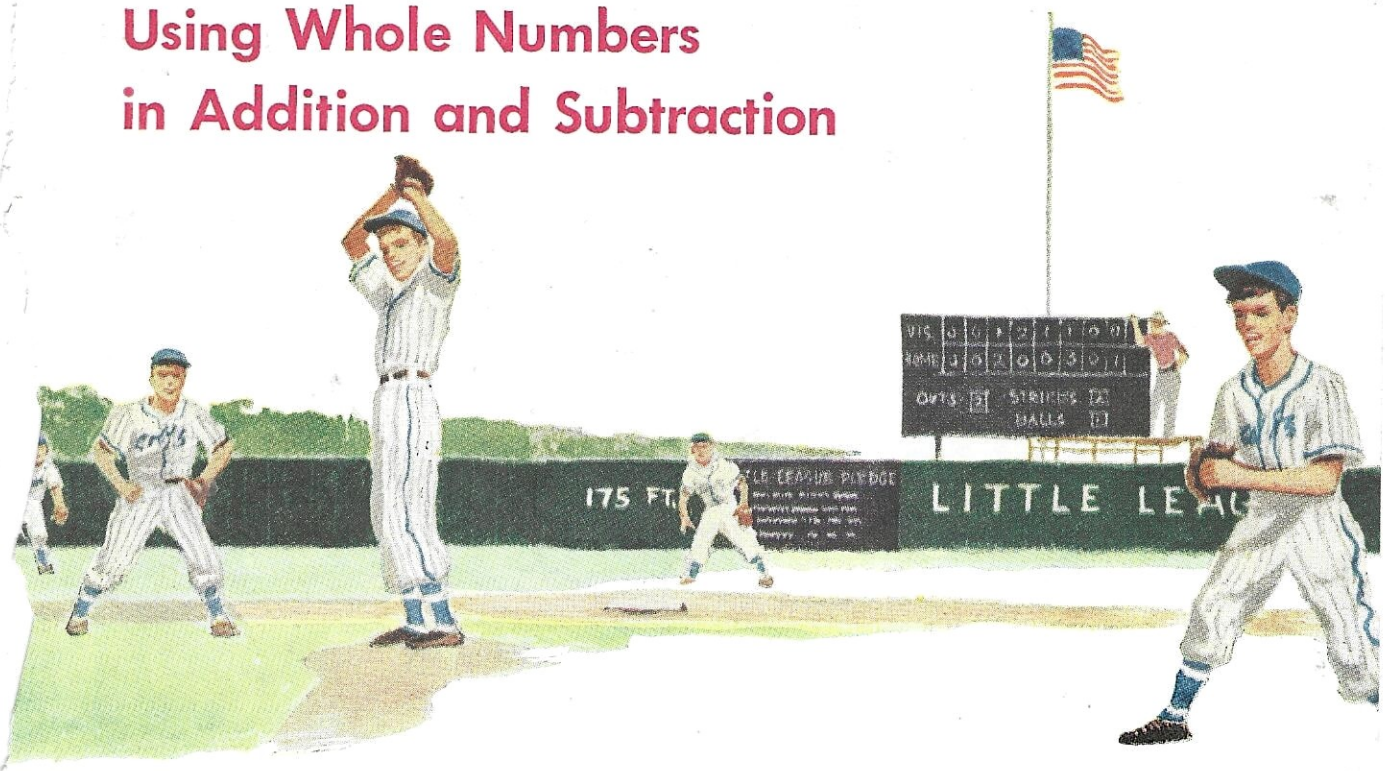
All of us know what to expect when we hear those words. For more than 100 years, baseball has been a favorite game of Americans of all ages.

Now there is a Little League for boys aged 8 to 12. The picture shows some of the players and some of the people who came to watch a Little League game.

Suppose that you are the sports reporter for your school paper and that you want to write the story of this game. Think of some of the ways you could count the people who came to watch.

1. You could count the people by ones. Can you say a number to go with each person? No matter how many people there are, will you ever run out of numbers? What does the number of the last person you count tell you?

Using Whole Numbers in Addition and Subtraction

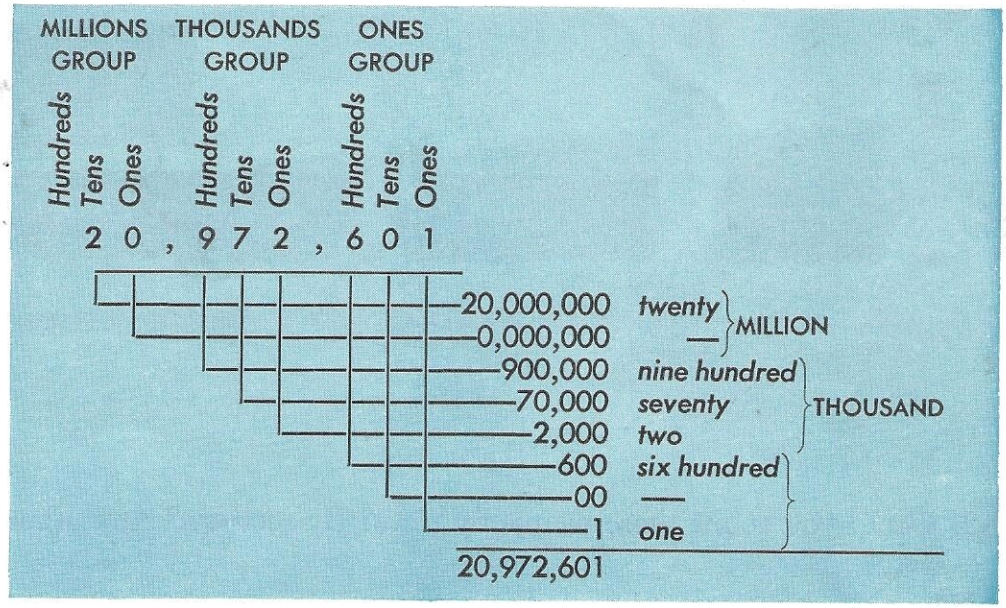


2. Could you count these people by rows? By sections? Any other way?
3. If the tickets are numbered and are sold in order, how can you tell how many people bought tickets? If the tickets are not numbered, but you know how many there were at first and how many were not sold, how can you tell how many tickets were sold?
4. Suppose a ticket says "Row 2, seat 7." Do those numbers tell how many or do they tell the position of the seat?
5. Use these sentences to show your understanding of the fact that numbers may tell *how many* or they may tell *order* or *position*:
   The Little Tigers have 15 boys in their club.
   Bob Young is number 15 on the club roll.
   The Tigers are playing their fifteenth game this week.
6. Does a number word like *fifteenth* always tell order or position? Does a number like 15 always tell how many?

Reading and Writing Large Numbers

Charles had a book about baseball. He read that 20,972,601 people attended major league games in a recent year.

The chart below will help you to read this number. Study the chart. Notice that you do not say anything for the zeros, and that you never say *and* in reading whole numbers.



There are commas which separate the figures into groups of three, beginning at the right. What is the same in each of the groups? What is different? If you know how to read numbers up to 999, you can read numbers of any size by just learning the names of the groups. 20,972,601 is read twenty million, nine hundred seventy-two thousand, six hundred one.

1. Read these numbers.

45,236,789	9,705,308	126,897,543	3,490,100
4,006,005	805,800	360,070,707	76,280

2. Write these numbers in figures.
   Nine thousand, six hundred seventy-three.
   Three hundred forty-five thousand, one hundred.
   Six million, three hundred thirty-five thousand.

For a puzzle, Helen needed to know the number of people in the whole world. In a reference book she found that the answer was 2,341,154,000.

You can read this number by learning that the group next larger than millions is the *billions* group.

Billions	Millions	Thousands	Ones
2,	341,	154,	000

The number is read 2 billion, 341 million, 154 thousand. It is a ten-place number. Count the figures in the number. Are there ten? What is the place value of the 2? Do you think you can read an eleven-place number now? A twelve-place number?

3. 451,360,297,433 is read _?_ billion, _?_ million, _?_ thousand, _?_.

4. 11,207,313,256 is read eleven _?_, two hundred seven _?_, three hundred thirteen _?_, two hundred fifty-six.

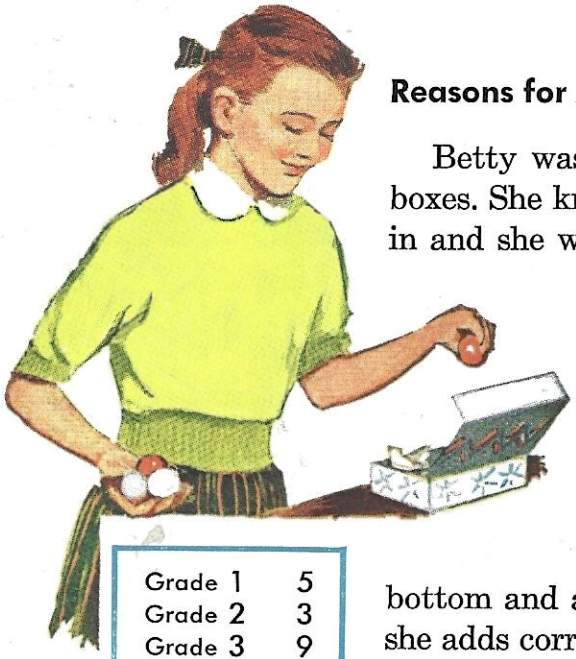
5. Make a chart like the one below and write the following numbers in the chart. What do the H, T, and O stand for?

Billions	Millions	Thousands	Ones
HTO	HTO	HTO	HTO
258	286	383	109

   (a) Two hundred fifty-eight billion, two hundred eighty-six million, three hundred eighty-three thousand, one hundred nine.
   (b) Forty-five billion, two hundred million, five hundred fifty-seven thousand, nine hundred eighty-two.
   (c) Three billion.
   (d) Two billion, two million, two thousand, two.

6. Tell the value of each of the figures you have written in your chart.

7. Which of the numbers in example 5 is a twelve-place number? Which is an eleven-place number? Which are ten-place numbers? What is the name of the group made up of the tenth, eleventh, and twelfth places?



Reasons for Adding; Signs; Terminology

Betty was helping Miss White with the Red Cross boxes. She knew how many balls each class had brought in and she wondered how many balls there were in all.

Betty knew that she could count all of the balls. She knew also that whenever numbers were *put together* in a problem, she could *add* to find the answer.

She wrote the numbers in a column as shown at the left. She added down, like this: 5, 8, 17, 23, 27, 35.

Grade 1	5
Grade 2	3
Grade 3	9
Grade 4	6
Grade 5	4
Grade 6	8
	35

To check her answer, she began at the bottom and added up, like this: 8, 12, 18, 27, 30, 35. If she adds correctly, she will get the same answer whether she adds up, down, or in any order.

Each number to be added is called an *addend*. There are six addends in this example. Name the six addends. The answer to an addition example is called the *sum*. What is the sum in this example? What does the sum tell you?

Betty could have written her example horizontally, like this: $5 + 3 + 9 + 6 + 4 + 8 = 35$.

The sign + means *add*. It is read *and* or *plus*. The sign = is called the *equal* sign. It may be read *equals*, *is*, or *are*.

For each of the following problems make a motion picture in your mind of the story the problem tells. Then answer these questions: Are numbers put together in the problem? Can you add to find the answer? What will the answer tell?

1. John had \$3.14. His father gave him \$5.00 for his birthday. How much did he have then?
2. The Little Tigers made 3 runs the first inning, 0 runs the second inning, 1 run the third inning, 5 runs the fourth inning, and 1 run the fifth inning. What was their total score?

Do You Know the Addition Facts?

INVENTORY TEST

You should be able to complete these facts without counting or other figuring of any kind. Write the sums on folded paper.

Make study cards for any facts that you miss. Practice these facts until you can make a perfect score on the test.

1.	3	4	2	9	6	2	1	7	4	0
	1	5	7	5	2	2	4	3	8	1
2.	1	5	1	4	4	9	7	2	1	0
	1	2	8	7	2	7	5	4	9	2
3.	6	8	8	1	5	8	7	5	6	0
	4	3	8	2	8	1	9	4	9	3
4.	3	8	3	5	7	9	4	3	6	0
	3	6	5	1	7	4	6	8	7	4
5.	2	8	8	6	9	7	5	1	3	0
	3	2	5	6	1	6	9	5	7	5
6.	2	9	2	4	6	8	1	7	8	0
	5	9	9	1	3	7	3	1	9	6
7.	3	2	3	5	7	2	1	6	1	0
	6	6	2	5	4	1	7	8	6	7
8.	7	5	6	3	2	7	3	4	4	0
	2	3	5	9	8	8	4	9	3	8
9.	6	9	5	8	9	9	5	4	9	0
	1	2	7	4	3	6	6	4	8	9
10.	1	2	3	4	5	6	7	8	9	0
	0	0	0	0	0	0	0	0	0	0

Camping at Deer Lake

2– AND 3–DIGIT ADDENDS, ZEROS, NO CARRYING

At camp last summer Charles and Robert collected rock specimens for the camp museum. Charles collected 18. Robert collected 20. How many did they have together?

Can you find how many they had together by counting? By adding? Is adding faster than counting if you know the addition facts?

18 means 1 ten 8 ones
20 means 2 tens 0 ones
38 means 3 tens 8 ones

First add the ones.

Then add the tens.

The boys had 38 specimens *together*.

The boys added their specimens to the 81 rocks already in the museum. How many specimens in all were in the museum then?

```
 38
 81
---
119
```

To find how many in all, *add*.

First add the ones. Write 9 in ones place.

Then add the tens. There are 11 tens. 11 tens make 1 hundred and 1 ten. Write 1 in hundreds place and 1 in tens place.

There were 119 specimens *in all* in the museum.

Add down. Check by adding up.

1.	32	53	23	36	40	54	83
	42	12	70	62	41	35	15
2.	84	54	36	55	73	23	65
	13	44	43	40	23	61	34
3.	20	70	10	61	20	26	30
	30	10	30	30	16	42	40
	40	10	50	13	23	30	10
4.	321	162	718	912	457	346	763
	106	230	211	180	132	121	233
5.	668	992	510	901	852	900	882
	821	403	988	828	824	951	411

Adding by Endings

WITHOUT AND WITH BRIDGING

1. You know $2 + 6 = 8$. $12 + 6 =$? $32 + 6 =$?
   Six added to any number ending in 2 gives a sum ending in 8 and does not change the tens figure. Use this fact to add in one step:
   $52 + 6$ $62 + 6$ $72 + 6$ $82 + 6$

2. You know $4 + 9 = 13$. Nine added to any number ending in 4 gives a sum ending in 3 and makes the tens figure larger by 1.
   $14 + 9 =$? $24 + 9 =$? $34 + 9 =$?
   $44 + 9 =$? $54 + 9 =$? $84 + 9 =$?
   Is $94 + 9$ more than 100? $94 + 9 =$?

The basic addition facts are *key facts* that help you do other additions. Use the key facts to give these sums in one step.

3. Read down the columns.

$5 + 8 = 13$	$7 + 8 = 15$	$7 + 9 = 16$
$15 + 8 = 23$	$7 + 18 = 25$	$27 + 9 =$?
$25 + 8 =$?	$7 + 28 =$?	$47 + 9 =$?
$35 + 8 =$?	$7 + 38 =$?	$67 + 9 =$?
$45 + 8 =$?	$7 + 48 =$?	$87 + 9 =$?

4.

23	57	31	65	5	81	73
+3	+2	+6	+3	+44	+7	+4

5.

19	46	5	77	85	63	73
+6	+8	+17	+6	+5	+7	+7

6.

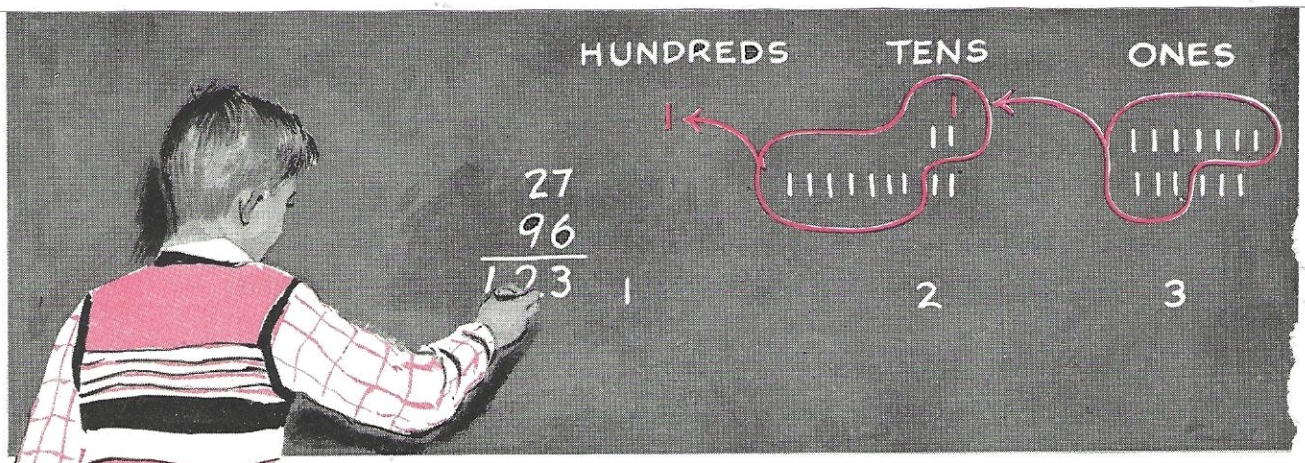
50	95	98	32	6	81	46
+6	+9	+9	+7	+25	+8	+6

Numbers ending in 0, 2, 4, 6, or 8 are *even* numbers.

7. If you add any two even numbers, will your sum be another even number?

Numbers ending in 1, 3, 5, 7, or 9 are *odd* numbers.

8. If you add two odd numbers, will your sum be an odd number?



Addition with Carrying

Sixth graders can add columns of numbers of any size because they know that our number system is based on tens. They know that 10 ones make 1 ten, 10 tens make 1 hundred, 10 hundreds make 1 thousand, and so forth.

1. Explain how knowing this helps them with the number they have to *carry* and in writing the answer for the example written on the blackboard at the top of this page.

2. See how the same ideas work in the example at the right.

$$\begin{array}{r} 311 \\ 295 \\ 242 \\ 289 \\ \hline 1137 \end{array}$$

First add the ones. Do you get 17? 17 ones may be thought of as 1 ten 7 ones. Write 7 in ones place and add the 1 ten with the other tens.

Next add the tens. Do you get 23? 23 tens may be thought of as 2 hundreds 3 tens. Write 3 in tens place and add the 2 hundreds with the other hundreds.

Then add the hundreds. Do you get 11? 11 hundreds may be thought of as 1 thousand 1 hundred. Write 1 in thousands place and 1 in hundreds place.

The answer is _?_. What do we call the answer to an addition example?

3. In the same way, explain how you add the following:

343	$37.98	$3.05	2901	4680	1,465,798
498	1.29	.75	3809	7320	8,652,313

4. Why can you add dollars and cents just as you add ordinary numbers?

Copy the following examples. Add down. Check by adding up.

5.

343	572	426	178	307	249	774
492	185	367	654	579	675	146

6.

715	384	356	824	406	782	501
682	533	582	323	672	400	469
494	496	614	706	583	338	330

7.

574	940	768	935	469	44	350
86	836	84	959	955	206	72
369	87	600	278	401	762	487
256	52	11	113	88	714	41

8.

4875	2136	7437	6739	3224	2995	2021
2017	4597	1895	5783	8998	6075	5679

9.

5327	3187	8259	9726	8753	2231	4500
7642	4592	7863	832	1204	728	206
7018	5716	6300	42	173	7483	816

10.

3	5	4	8	70	69	60	47
4	2	9	3	25	17	50	86
1	4	7	7	39	20	44	24
5	3	6	8	60	46	49	58
2	5	0	2	44	18	20	73
1	8	8	9	36	59	40	89

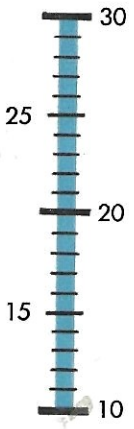
11.

$1850.00	$6009.05	$7800.10	$1500.00	$ 783.08
868.79	300.85	25.00	250.00	2087.09

12.

$.01	$.07	$.08	$3.65	$2.70	$3.73
.03	.24	.56	2.03	3.64	2.24
.02	.15	.77	1.34	1.17	4.04
.03	.10	.30	4.72	1.00	1.06

Rounding Numbers



The Little Tigers are going to play a game in Middletown. The coach says that Middletown is about 20 miles away. When he says *about 20 miles*, he is using an approximate number.

1. May the actual distance be more than 20 miles? Less than 20 miles?

2. What number is halfway between 10 and 20? 20 and 30? How do you tell whether a number is nearer to 20 than to 10? Nearer to 20 than to 30? Any number larger than 15 and smaller than 25 may be rounded to _?_.

A number halfway between two tens is not really nearer to one than the other. Usually we round such numbers to the next higher ten, but we often call it rounding to the nearest ten. We say that 55 rounded to the nearest ten is 60.

3. Round these numbers to the nearest ten:
   82 63 78 41 25 29 56 37 35 94

4. Use the same kind of thinking to round these air distances to the nearest hundred miles:

Washington to Atlanta	547 mi.
Cleveland to Chicago	307 mi.
Pittsburgh to Boston	483 mi.
Chicago to Boston	879 mi.
Memphis to Detroit	635 mi.
Cincinnati to Houston	933 mi.

5. Round these air distances to the nearest 1000 miles:

Boston to Jacksonville	1,023 mi.
Kansas City to Miami	1,342 mi.
Los Angeles to Memphis	1,666 mi.
Los Angeles to Washington	2,331 mi.
Los Angeles to Jacksonville	2,657 mi.
El Paso to New Orleans	992 mi.

Whole numbers may be rounded to any place, and rounding is always done by the same kind of thinking. Mary has a trick to help her. She puts an arrow above the figure in the place to which she is rounding. If the figure to the right of the arrow figure is 5 or more, she makes the arrow figure 1 larger. If the figure to the right of the arrow figure is less than 5, she leaves the arrow figure as is. Then she puts zeros in all the places to the right of the arrow figure. Do you see why Mary's trick works?

Study the following example.

$\overset{\downarrow}{7},142,685$ rounded to the nearest million is 7,000,000.

$7,\overset{\downarrow}{1}42,685$ rounded to the nearest hundred thousand is 7,100,000.

$7,1\overset{\downarrow}{4}2,685$ rounded to the nearest ten thousand is 7,140,000.

$7,14\overset{\downarrow}{2},685$ rounded to the nearest thousand is 7,143,000.

$7,142,\overset{\downarrow}{6}85$ rounded to the nearest hundred is 7,142,700.

$7,142,68\overset{\downarrow}{5}$ rounded to the nearest ten is 7,142,690.

6. Do you think it would often be useful to round a number larger than 7 million to the nearest hundred or ten? Usually we round numbers as large as this so that all the figures are zeros except the first one or two figures.

7. Sometimes round numbers are written using words instead of some of the zeros. Write the numbers in the following sentences using only figures.
   (a) There are more than 2 billion people in the world.
   (b) Our new school cost $\$\frac{1}{2}$ million. ($\frac{1}{2}$ million = _?_ thousand)
   (c) My father paid 28 hundred dollars for his car.
   (d) About 343 thousand people live in our city.
   (e) It will take $\$1\frac{1}{2}$ million to widen this highway.

Estimating Answers

ADDITION

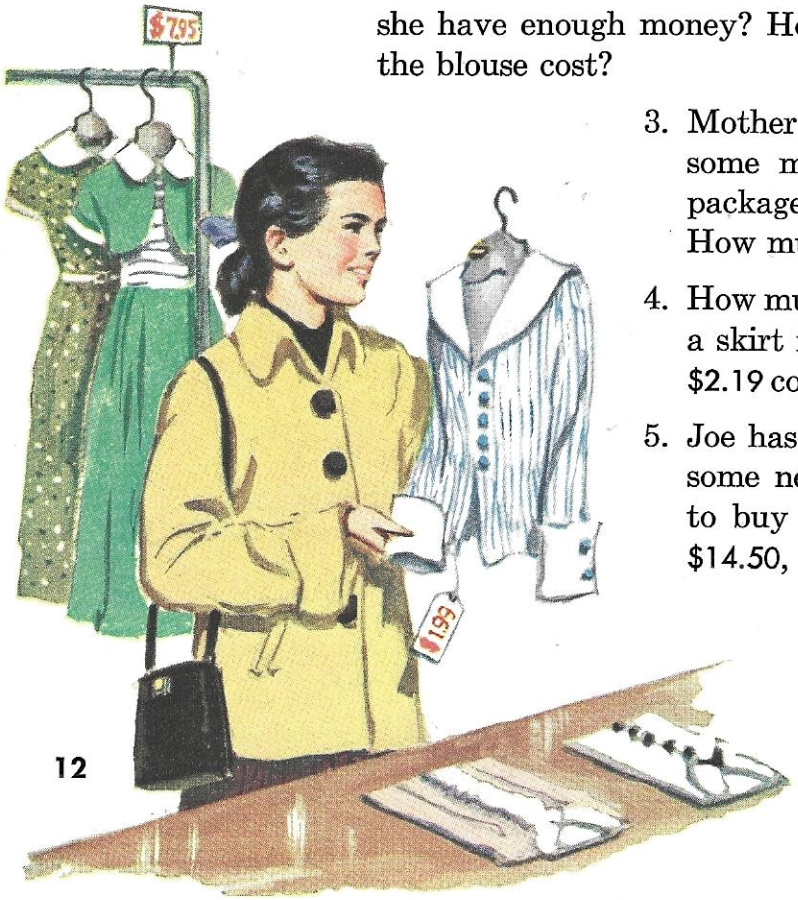
Estimated Answer
40¢
50¢
90¢
Exact Answer
38¢
53¢
91¢

Alice Smith's mother sent her to the store to get a pint of cream that cost 38¢ and a dozen eggs that cost 53¢. How much did they both cost?

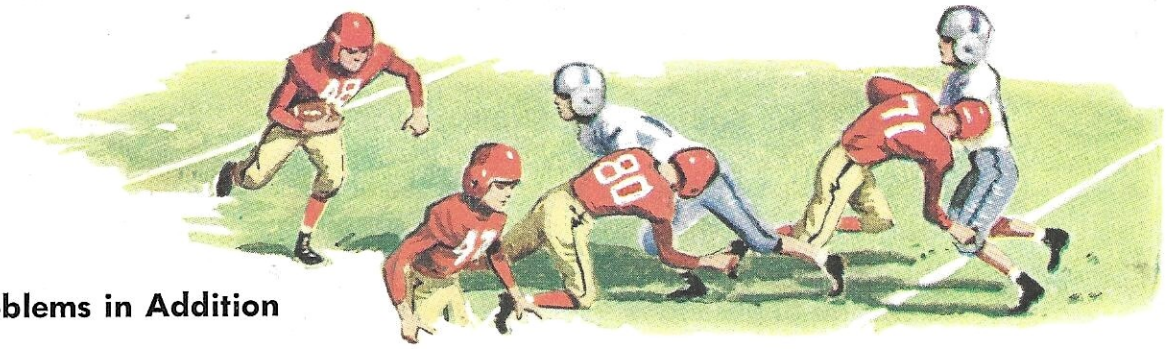
First Alice estimated the answer using round numbers. Alice thought, 38 is about 40; 53 is about 50; 40¢ and 50¢ are 90¢. The answer is about 90¢. Then she found the exact answer. Is it about 90¢? Estimating is a good way to be sure your answer is sensible.

For each of the problems below, estimate the answer by rounding prices to the nearest dollar. Then find the exact answer.

1. John's father bought John a sport coat that cost $11.50 and slacks that cost $4.25. How much did he spend?
2. Ann received $10 for her birthday. She wants to buy a dress that costs $7.95 and a blouse that costs $1.99. Will she have enough money? How much will the dress and the blouse cost?



3. Mother sent Bob to the store for some meat which cost 83¢ and a package of cheese which cost $1.24. How much did Bob spend?
4. How much would a dress for $16.85, a skirt for $3.98, and a sweater for $2.19 cost?
5. Joe has $25.00 that he saved to buy some new clothes. Will he be able to buy slacks for $4.25, a suit for $14.50, and a jacket for $6.95?
6. What is the sum of $.95, $2.07, $5.63, and $8.29?



Problems in Addition

Read each problem carefully. See what question is asked. Is the problem about putting together? If so, add to find the answer. For each problem, first estimate the answer, then find the exact answer. Always check your work.

1. How much did Calvert School spend for football equipment if it spent $75.60 for pants, $60.00 for shoes, $47.50 for helmets, $87.90 for sweaters, $9.40 for stockings, and $24.00 for footballs? (Round prices to the nearest ten dollars for estimating.)
2. What is the total number of children in the six grades of Akin School if there are 30 children in first grade, 36 in second, 37 in third, 34 in fourth, 36 in fifth, and 33 in sixth?
3. The Alaska Highway runs from Dawson Creek in Canada to Fairbanks, Alaska. It is 1221 miles from Dawson Creek to the Canada-Alaska border and 302 miles from there to Fairbanks. How long is the Alaska Highway?
4. In a hospital for pets there were 17 dogs, 11 cats, 3 birds, and 1 turtle. In all, how many pets were in the hospital?
5. In doing his homework, Jack spent 20 minutes on arithmetic, 12 minutes on spelling, 25 minutes on social studies, and 30 minutes drawing a picture for science. How many minutes did Jack spend on his homework? Is that more than an hour?
6. How many yards of wool string are used in making a baseball if 121 yards of rough gray wool string, 45 yards of white wool string, and 53 yards of fine gray wool string are used?
7. A mining company had 1,208 men working in the mine, 1,004 men working in the mill, and 376 people in offices. How many people did they employ?

Reasons for Subtracting; Signs; Terminology

If you learn to see in your mind the action that a problem describes, you will know when to subtract.

Addition and subtraction are opposite processes. You may add to put numbers together; you may subtract to take numbers apart.

There are two basic problem situations in which you may subtract.

1. You know a number that tells how many or how much there is in all. You know one part of this number, and you want to find the other part.

The question asks something which means: How many are left? How many are gone? How many are of one kind?

2. You know two numbers which you wish to compare.

Some of the ways the question may be worded are: How many more? How much more? How many fewer? How much less? What is the difference? How much older? How much taller? How much later? How much more is needed? How many more are needed?

The sign — means *subtract*. 8 — 3 may be read 8 take away 3, or 8 minus 3, or 8 subtract 3, or 3 from 8.

Look at the example to review what the three numbers in a subtraction example are called. The answer may be called either the *difference* or the *remainder*, but *difference* is used more often.

57 minuend
38 subtrahend
19 difference

Do You Know the Subtraction Facts?

INVENTORY TEST

You should be able to complete these facts without counting or other figuring of any kind. Write the answers on folded paper.

Make study cards for any facts that you miss. Practice these facts until you can make a perfect score on the test.

1.	7	3	11	13	16	5	12	6	8	1
	5	1	9	4	8	2	8	3	3	0
2.	4	9	7	11	8	10	9	6	16	2
	3	3	6	4	5	6	8	4	7	0
3.	3	15	6	10	11	9	10	14	12	3
	2	6	5	7	3	4	1	7	5	0
4.	9	13	10	15	2	16	10	6	10	4
	6	9	8	9	1	9	2	2	5	0
5.	4	13	7	13	9	14	12	14	14	5
	2	8	3	7	2	8	6	9	5	0
6.	9	11	17	4	13	10	8	7	12	6
	1	7	9	1	6	4	7	4	4	0
7.	12	18	8	11	14	9	5	15	12	7
	7	9	6	5	6	5	4	8	3	0
8.	11	8	9	8	11	5	10	12	17	8
	6	1	7	2	2	1	3	9	8	0
9.	10	15	11	7	7	13	5	8	6	9
	9	7	8	1	2	5	3	4	1	0
10.	11	12	13	14	15	16	17	18	10	0
	0	0	0	0	0	0	0	0	0	0

Subtracting Two- and Three-Place Numbers

ZEROS; NO EXCHANGE

If you know the subtraction facts, you can do the following examples very quickly. In subtracting larger numbers, begin at the right. Subtract the ones first, then the tens, and so forth.

To check subtraction, add the difference and the subtrahend. If the work is correct, their sum will be the same as the minuend. You should be able to do the checking in your head, just by adding up the two bottom numbers in the example and seeing if you get the top number.

99 Minuend	Check:	81 (Subtrahend)	Check:	99	
−81 Subtrahend		+18 (Difference)		−81	} Add up
18 Difference		99 (Minuend)		18	

Subtract and check.

1.	99	83	76	54	45	67	88
	73	12	30	11	20	51	32
2.	89	57	89	78	89	74	98
	52	56	43	58	64	24	50
3.	599	257	290	549	359	147	780
	306	120	270	408	309	137	470
4.	143	125	157	124	169	134	149
	73	65	87	72	83	62	87
5.	116	168	138	109	149	112	120
	66	97	88	79	50	50	80
6.	186	135	159	101	110	175	139
	92	90	63	80	40	85	89
7.	593	635	726	298	436	388	999
	281	413	615	177	413	156	654

Subtracting by Endings

$\begin{array}{r} 8 \\ -6 \\ \hline 2 \end{array}$ $\begin{array}{r} 18 \\ -6 \\ \hline 12 \end{array}$ $\begin{array}{r} 28 \\ -6 \\ \hline 22 \end{array}$ $\begin{array}{r} 38 \\ -6 \\ \hline 32 \end{array}$ $\begin{array}{r} 48 \\ -6 \\ \hline 42 \end{array}$ $\begin{array}{r} 58 \\ -6 \\ \hline 52 \end{array}$ $\begin{array}{r} 68 \\ -6 \\ \hline 62 \end{array}$ $\begin{array}{r} 78 \\ -6 \\ \hline 72 \end{array}$ $\begin{array}{r} 88 \\ -6 \\ \hline 82 \end{array}$ $\begin{array}{r} 98 \\ -6 \\ \hline 92 \end{array}$

$\begin{array}{r} 15 \\ -8 \\ \hline 7 \end{array}$ $\begin{array}{r} 25 \\ -8 \\ \hline 17 \end{array}$ $\begin{array}{r} 35 \\ -8 \\ \hline 27 \end{array}$ $\begin{array}{r} 45 \\ -8 \\ \hline 37 \end{array}$ $\begin{array}{r} 55 \\ -8 \\ \hline 47 \end{array}$ $\begin{array}{r} 65 \\ -8 \\ \hline 57 \end{array}$ $\begin{array}{r} 75 \\ -8 \\ \hline 67 \end{array}$ $\begin{array}{r} 85 \\ -8 \\ \hline 77 \end{array}$ $\begin{array}{r} 95 \\ -8 \\ \hline 87 \end{array}$ $\begin{array}{r} 105 \\ -8 \\ \hline 97 \end{array}$

Dick says that all the basic subtraction facts are *key facts* that help in doing other subtractions. Study Dick's chart. Do you see why he calls it "Subtracting by Endings"?

1. Tell nine other facts that each of the following facts helps you with: $5 - 2 = 3$ $11 - 6 = 5$.

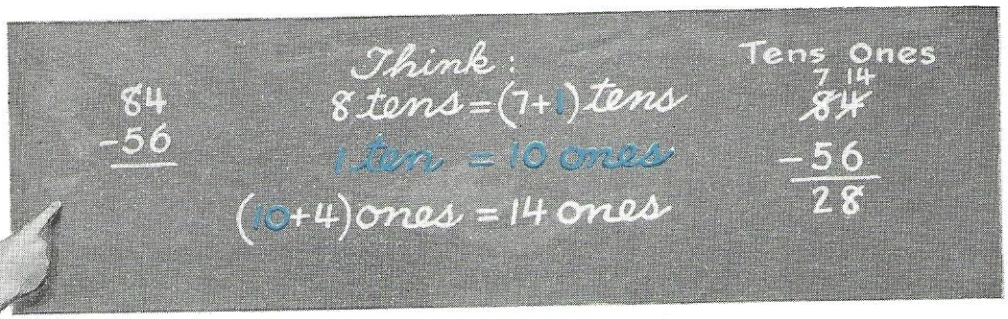
Use the key facts. Practice the following subtractions until you can do them correctly in one step.

2.	$\begin{array}{r} 17 \\ -6 \\ \hline \end{array}$	$\begin{array}{r} 37 \\ -6 \\ \hline \end{array}$	$\begin{array}{r} 47 \\ -6 \\ \hline \end{array}$	$\begin{array}{r} 19 \\ -5 \\ \hline \end{array}$	$\begin{array}{r} 39 \\ -5 \\ \hline \end{array}$	$\begin{array}{r} 49 \\ -5 \\ \hline \end{array}$
3.	$\begin{array}{r} 24 \\ -4 \\ \hline \end{array}$	$\begin{array}{r} 94 \\ -4 \\ \hline \end{array}$	$\begin{array}{r} 67 \\ -7 \\ \hline \end{array}$	$\begin{array}{r} 88 \\ -7 \\ \hline \end{array}$	$\begin{array}{r} 57 \\ -3 \\ \hline \end{array}$	$\begin{array}{r} 69 \\ -6 \\ \hline \end{array}$
4.	$\begin{array}{r} 79 \\ -9 \\ \hline \end{array}$	$\begin{array}{r} 99 \\ -9 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ -1 \\ \hline \end{array}$	$\begin{array}{r} 89 \\ -8 \\ \hline \end{array}$	$\begin{array}{r} 77 \\ -2 \\ \hline \end{array}$	$\begin{array}{r} 79 \\ -7 \\ \hline \end{array}$
5.	$\begin{array}{r} 68 \\ -9 \\ \hline \end{array}$	$\begin{array}{r} 78 \\ -9 \\ \hline \end{array}$	$\begin{array}{r} 98 \\ -9 \\ \hline \end{array}$	$\begin{array}{r} 57 \\ -8 \\ \hline \end{array}$	$\begin{array}{r} 67 \\ -8 \\ \hline \end{array}$	$\begin{array}{r} 97 \\ -8 \\ \hline \end{array}$
6.	$\begin{array}{r} 66 \\ -8 \\ \hline \end{array}$	$\begin{array}{r} 76 \\ -8 \\ \hline \end{array}$	$\begin{array}{r} 96 \\ -8 \\ \hline \end{array}$	$\begin{array}{r} 36 \\ -9 \\ \hline \end{array}$	$\begin{array}{r} 46 \\ -9 \\ \hline \end{array}$	$\begin{array}{r} 56 \\ -9 \\ \hline \end{array}$
7.	$\begin{array}{r} 15 \\ -6 \\ \hline \end{array}$	$\begin{array}{r} 25 \\ -6 \\ \hline \end{array}$	$\begin{array}{r} 75 \\ -6 \\ \hline \end{array}$	$\begin{array}{r} 85 \\ -6 \\ \hline \end{array}$	$\begin{array}{r} 95 \\ -6 \\ \hline \end{array}$	$\begin{array}{r} 105 \\ -6 \\ \hline \end{array}$

8.	$26 - 9$	$34 - 6$	$25 - 7$	$65 - 7$
9.	$88 - 9$	$53 - 9$	$61 - 7$	$45 - 7$
10.	$56 - 8$	$97 - 9$	$43 - 5$	$55 - 7$

Subtraction with Exchange

Every number may be thought of in many different ways. For example, 99 may be thought of as $90 + 9$ (the usual way), as $100 - 1$, as $98\frac{4}{4}$, as $80 + 19$, and in many other ways. We decide which way to think of a number by what we want to do with it.



Carol is showing the class how to subtract 56 from 84. She cannot subtract 6 ones from 4 ones. She uses the idea that 1 ten = 10 ones to change the form of the 84 from 8 tens 4 ones to 7 tens 14 ones. Now she can subtract 6 ones from 14 ones, leaving 8 ones, and 5 tens from 7 tens, leaving 2 tens. The *difference* is 28. Carol says that she really does in her mind the changes she has shown on the blackboard. Changing the form of the minuend in this way is sometimes called *exchanging* or *regrouping* in subtraction.

1. Use Carol's idea to explain how you subtract 627 from 945.

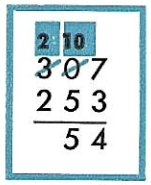
Copy and subtract. Check your work.

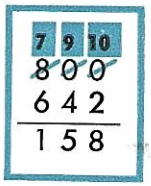
2.	691 529	984 218	698 319	895 238	782 629	573 219	451 328
3.	931 817	973 758	986 627	577 419	492 347	882 258	684 539
4.	775 247	594 367	696 479	793 357	997 828	785 349	676 428

Study how the form of the minuend is changed in the following examples:

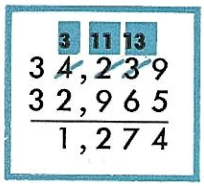
(a) Subtract 253 from 307. Since 1 hundred = 10 tens, 307 may be thought of as 2 hundreds 10 tens 7 ones. Then the subtraction is easy.

(b) Subtract 642 from 800. Why may 800 be thought of as 7 hundreds 9 tens 10 ones?





5. Does every place in our number system have ten times the value of the next place to its right? Use this idea to explain how you subtract 32,965 from 34,239.



Copy and subtract. Always check your work.

6.	406	709	805	403	502	908
	234	463	182	371	242	197
7.	800	500	700	400	600	300
	154	326	468	125	132	189
8.	200	400	300	900	800	200
	165	389	278	815	799	198
9.	7351	6452	8135	9363	7427	
	2675	1676	3768	4478	5749	
10.	32,211	59,123	79,235	88,572	65,784	
	21,572	38,456	38,670	27,968	34,776	
11.	24,432	35,321	48,726	79,328	108,563	
	14,678	25,436	38,839	69,659	98,678	

12. Bob wanted to know whether Fenway Park in Boston or Comiskey Park in Chicago could hold more fans at a baseball game. He read that Fenway Park could seat 34,824 persons and that Comiskey Park could seat 46,550 persons. Which park could seat more? How many more?

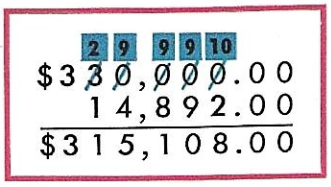
Subtracting Numbers of Dollars and Cents

Dollars and cents are subtracted like ordinary numbers because our way of writing dollars and cents is based on ten, like our number system.

Subtract and check. Do not forget the dollar sign and decimal point in your answers.

1.	\$2.87	\$3.57	\$12.75	\$15.25	\$25.40
	.05	1.04	6.29	9.48	12.15
2.	\$9.07	\$8.07	\$22.16	\$47.15	\$50.47
	4.56	3.56	10.58	24.67	33.58
3.	\$14.00	\$9.00	\$43.00	\$60.00	\$79.50
	7.25	3.75	16.54	33.48	58.07
4.	\$559.43	\$123.00	\$820.00	\$703.50	\$683.60
	207.37	57.69	416.24	455.86	644.08

Business firms and government agencies often need very large numbers in keeping their records. Sometimes they have to do subtractions with several zeros in the minuend. Changing the form of the minuend in these examples is easy if you pretend that you have a supply of coins and bills. Think how you would change the money, tens for ones, hundreds for tens, and so forth.



Study the example at the left. Notice that you need 10 ones and that you need 9 in every other place until you come to the place to the left of the zeros.

Subtract and check.

5.	\$28,000.00	\$39,000.00	\$80,000.00	\$70,000.00
	7,891.00	8,623.00	54,321.00	34,965.00
6.	\$200,000.00	\$400,000.00	\$700,000.00	\$600,000.00
	78,962.00	265,439.00	560,899.00	598,238.00

7.

$60,000,000.00	$80,000,000.00	$90,000,000.00
41,234,069.89	56,759,855.05	88,899,991.05

Examples like those above seldom occur in real-life problems that do not involve money. A number like 90 million is usually a round number, and it would not make sense to subtract an exact number from a round number. Numbers used in examples should either all be exact or all be rounded to the same place.

Helps in Addition and Subtraction

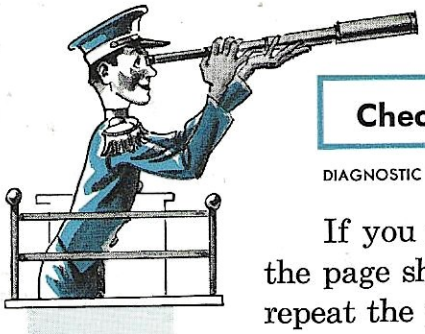
Make up several examples that will show how each of the following ideas may help you with the addition and subtraction facts.

For addition:

1. Changing the order of the addends does not change the sum. For example, $8 + 7 = 15$ and $7 + 8 = 15$. If you know the sum of one combination, you know its reverse.
2. Any number added with 1 gives the next higher number.
3. Any number added with 0 gives that number.
4. You can use facts you know to help you with facts you are not sure of. For example, if you know $6 + 6 = 12$, you can tell that $7 + 6$ must be 1 more than 12, or 13.

For subtraction:

5. One subtracted from any number gives the next smaller number.
6. Zero subtracted from any number gives that number.
7. Any number subtracted from itself gives zero.
8. Addition will help you with subtraction. For example, if you know $8 + \_?\_ = 15$, you can subtract $15 - 8$.
9. If you have other ideas that are useful to you, explain your ideas to the other pupils in your group.



Checking Up on Addition and Subtraction

DIAGNOSTIC TEST ON PP. 6–20

If you make mistakes in any row of examples, turn to the page shown beside the row. Study the explanation and repeat the practice. Then try the row on this page again.

Add down. Check by adding up.

6

1.

32	53	36	55	457	900	862
42	12	43	40	132	951	431

8

2.

715	384	356	935	469	44	350
682	533	582	959	955	206	72
494	496	614	278	401	762	487
			113	88	714	41

8

3.

4875	2136	7437	9726	8753	2231	4500
2017	4597	1895	832	1204	728	206
			42	173	7483	816

8

4.

3	5	4	8	70	69	60	47
4	2	9	3	25	17	50	86
1	4	7	7	39	20	44	24
5	3	6	8	60	46	49	58
2	5	0	2	44	18	20	73
1	8	8	9	36	59	40	89

8

5.

2317	7364	5679	79,897	39,008
7891	4283	9743	75,525	27,016
		7868	47,978	89,120
			64,434	97,003

9

6.

$.01	$3.65	$23.40	$37.05	$39.04	$ 3.09
.03	2.03	15.25	22.03	.65	10.24
.02	1.34	11.50	6.04	1.27	7.00
.03	4.72	21.32	5.06	2.02	.75

Subtract and check.

7.	99	83	76	138	298	824	296	16
	73	12	30	88	177	724	192	
8.	691	973	986	577	997	882	676	18
	529	758	627	419	828	258	428	
9.	908	800	400	8135	79,235			19
	197	154	125	3768	38,670			
10.	$3.95	$3.49	$43.00	$683.42	$512.09			20
	.07	1.04	17.25	286.91	317.90			

Mathematical Sentences

ADDITION AND SUBTRACTION OF WHOLE NUMBERS

Mathematical sentences can be read when you know the meaning of symbols. You know the meaning of +, −, and = in addition and subtraction of whole numbers. Symbols such as □, △, n, and _?_ ask you to find the missing numbers in mathematical sentences. For example: $7 + \square = 9$, $7 + \triangle = 9$, $7 + n = 9$, and $7 +$ _?_ $= 9$ ask you to find a missing number. What is the missing number in these four sentences?

Copy the following sentences and write the whole number that will make each sentence true.

1.	$34 + \square = 84$	$26 + \triangle = 52$	$14 + n = 38$
2.	$205 +$ _?_ $= 260$	$75 - n = 25$	$n - 1423 = 236$
3.	$n + 1240 = 2460$	$134 - \triangle = 70$	$\square - 64 = 138$

When the same symbol is used more than once in the same sentence, it stands for the same number. For example, the boxes in $\square + \square = 20$ both stand for 10 if the sentence is true. Write this example on your paper as $10 + 10 = 20$.

Write the number that makes each sentence true.

4.	$\square + \square = 60$	$48 - n = n$	$\triangle + \triangle = 128$
5.	$90 - \square = \square$	$n + n = 250$	$\square + \square + \square = 273$
6.	$15 + n = 2 \times n$	$56 - \triangle = \triangle + 30$	$360 - \square - \square = 120$

Can You Do This?

ENRICHMENT PAGE

1. On a radio quiz program Jack heard a performer tell how he added columns of numbers very quickly. Here is his example and his method.

48
56
32
87

First add 48 to the tens in 56; then add the ones. Continue in a similar way with the other numbers, from top to bottom, as follows:

48 + 50 = 98; 98 + 6 = 104; 104 + 30 = 134;
134 + 2 = 136; 136 + 80 = 216; 216 + 7 = 223

Add these columns the quick way. Check by another method.

(a)	(b)	(c)	(d)	(e)
46	55	24	53	38
33	42	36	24	83
57	63	72	66	62
75	81	49	42	78

2. Add 9 to a number. Compare the sum of the figures in your answer with the sum of the figures in the original number. For example: 142 + 9 = 151. In the 142, 1 + 4 + 2 = 7; in the 151, 1 + 5 + 1 = 7. Try some other examples. What do you find out? Can you explain why this is true?
3. Subtract 9 from a number. Compare the sum of the figures in your answer with the sum of the figures in the original number. Try several examples. What do you find out? Can you explain why this is true?
4. Look back at exercises 2 and 3. Do you think the sum of the figures in the answer would be equal to the sum of the figures in the original number if you added or subtracted any one-figure number except 9?
5. Try the same thing you did in exercises 2 and 3, adding and subtracting two-figure numbers that are multiples of 9, such as 18 and 27. Then when any sums have more than one figure, add the figures in those sums and compare the final one-figure totals.

Roman Numerals

Think of some of the places where you have seen Roman numerals in use. Often you wish to read the date on a building, a movie film, or a monument that is dated in Roman numerals.

In reading these dates, there are three rules about Roman numerals that you must know.

I =	1
V =	5
X =	10
L =	50
C =	100
D =	500
M =	1000

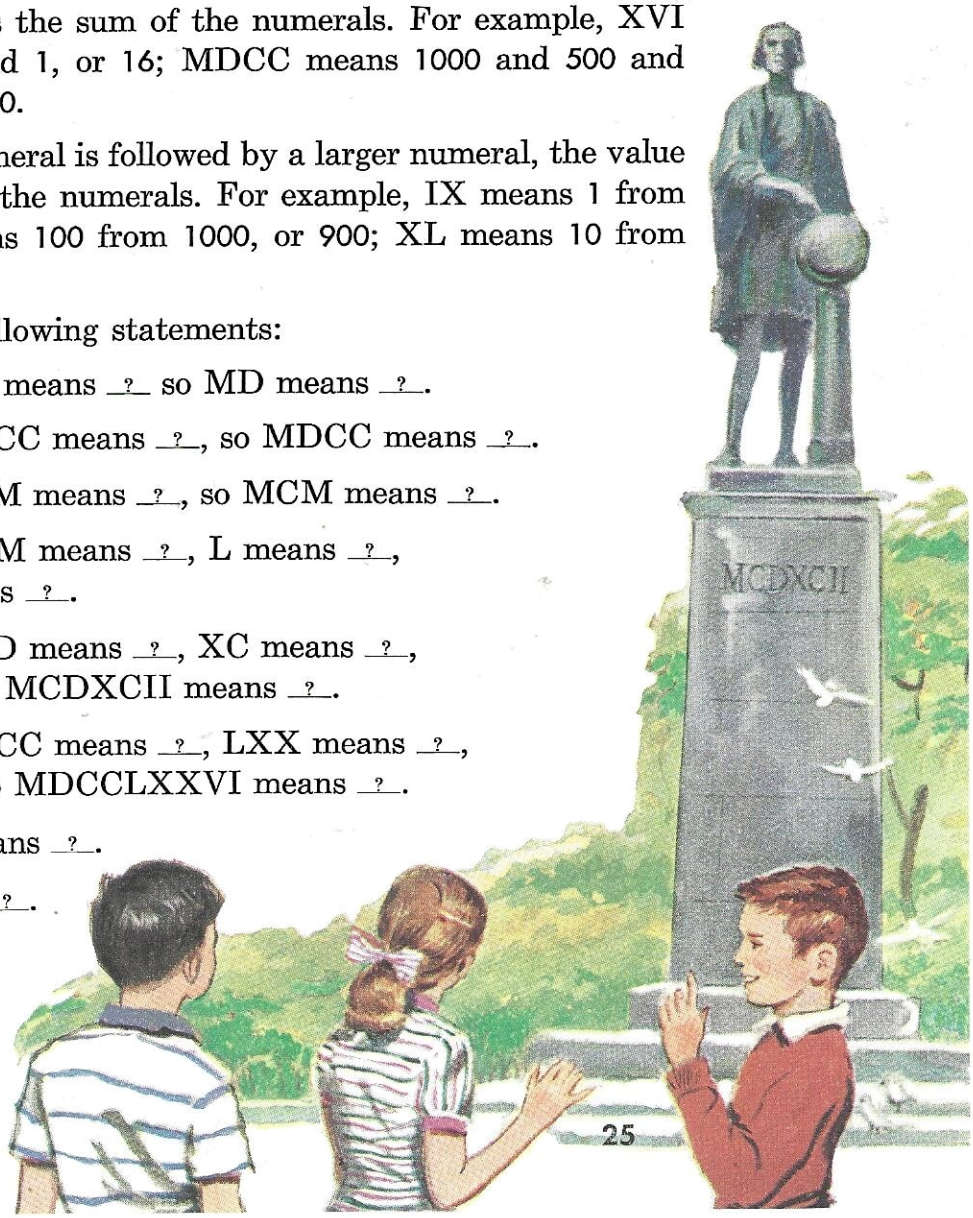
I. When a numeral is repeated, its value is repeated. For example, XX means 10 and 10, or 20; CCC means 300.

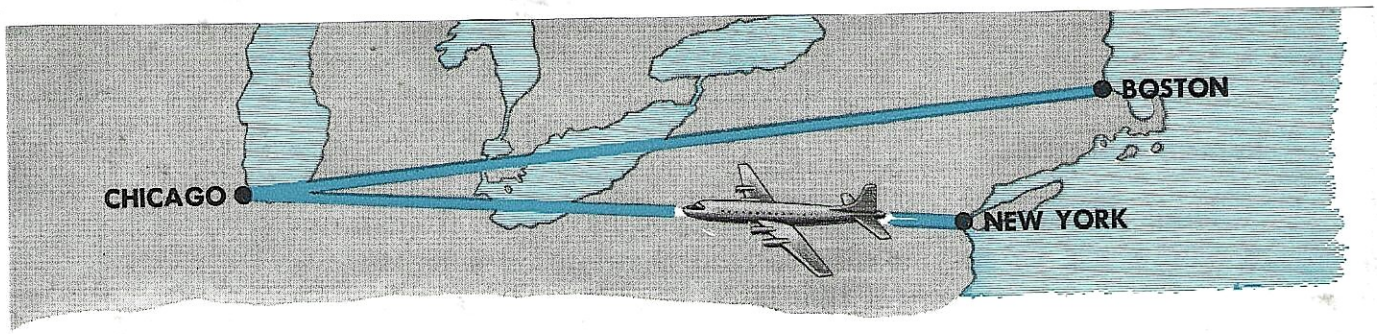
II. When a numeral is followed by one or more smaller numerals, the value is the sum of the numerals. For example, XVI means 10 and 5 and 1, or 16; MDCC means 1000 and 500 and 100 and 100, or 1700.

III. When a numeral is followed by a larger numeral, the value is the difference of the numerals. For example, IX means 1 from 10, or 9; CM means 100 from 1000, or 900; XL means 10 from 50, or 40.

Complete the following statements:

1. M means _?_, D means _?_ so MD means _?_.
2. M means _?_, DCC means _?_, so MDCC means _?_.
3. M means _?_, CM means _?_, so MCM means _?_.
4. M means _?_, CM means _?_, L means _?_, so MCML means _?_.
5. M means _?_, CD means _?_, XC means _?_, II means _?_, so MCDXCII means _?_.
6. M means _?_, DCC means _?_, LXX means _?_, VI means _?_, so MDCCLXXVI means _?_.
7. MCMLVIII means _?_.
8. MMLX means _?_.





Estimating Answers

SUBTRACTION

While studying aeronautics in science, the sixth graders became interested in the air distances between certain large cities. John found out that the distance by air from Boston to Chicago was 879 miles and from New York to Chicago was 724 miles. He wondered how much farther it was from Boston to Chicago than from New York to Chicago.

$$\begin{array}{r} 879 \\ -724 \\ \hline 155 \end{array}$$

First John estimated his answer. He said: "879 is about 880 and 724 is about 720. $880 - 720 = 160$." Then he found the exact answer. Is the exact answer near the estimated answer?

Estimating answers is useful in all kinds of problems. If you are good at estimating, it will help you to tell whether your answers are sensible.

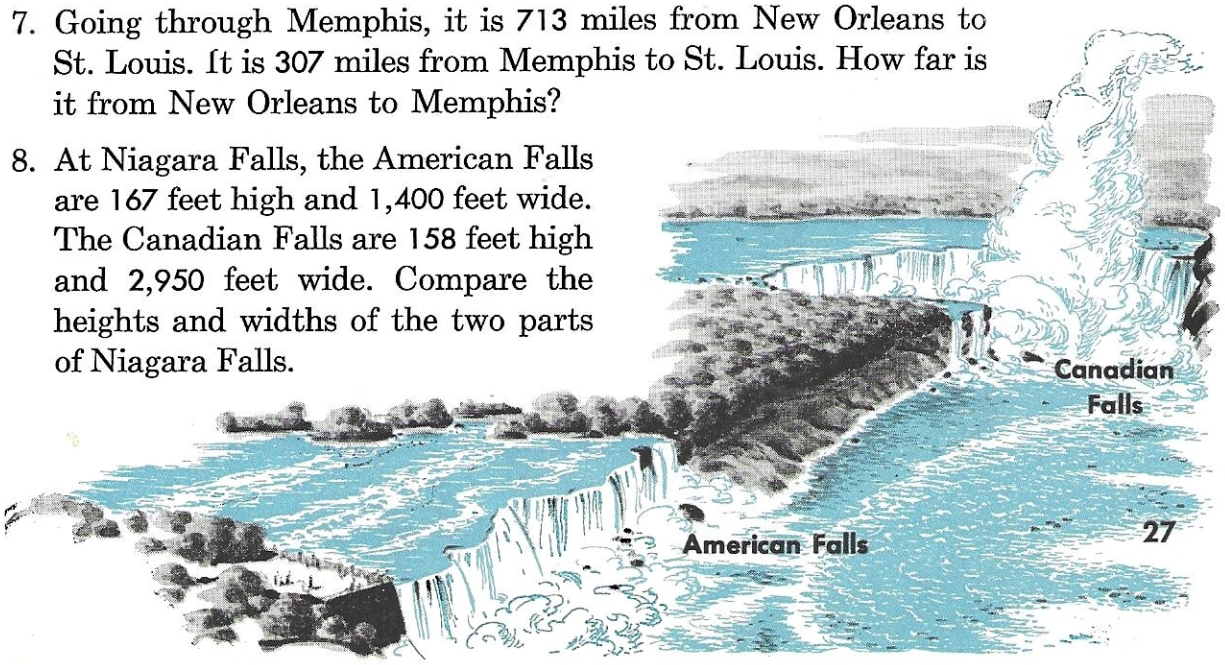
For each of the sets of air distances given below, make up a question that can be answered by subtraction. Estimate the answer to each of your questions by rounding distances to the nearest ten miles. Then find the exact answer.

1. The distance between Atlanta, Georgia and San Francisco is 2,308 miles and between Atlanta and Seattle, Washington is 2,354 miles.
2. The distance between St. Louis, Missouri and Dallas, Texas is 547 miles and between St. Louis and Denver, Colorado is 781 miles.
3. From Miami, Florida to Cleveland, Ohio is 1,109 miles and from Miami to Indianapolis, Indiana is 1,040 miles.
4. The distance from Detroit, Michigan to Louisville, Kentucky is 333 miles and from Detroit to Memphis, Tennessee is 635 miles.

Problems in Subtraction

Read each problem carefully. See what question is asked. Is the problem about finding one of two parts of a number or about comparing two numbers? If so, subtract to find the answer. For each problem, first estimate the answer, then find the exact answer. Always check your work.

1. How much change would John receive from a ten-dollar bill if he bought a sweater for $2.95?
2. In professional baseball the bases are 90 feet apart. In Little League Baseball the bases are 60 feet apart. What is the difference of these distances?
3. Babe Ruth hit a total of 714 home runs. Jimmy Foxx hit a total of 534 home runs. How many more home runs did Ruth hit than Foxx?
4. The area of New York state is 49,576 square miles. The area of New Mexico is 121,666 square miles. How much larger is New Mexico than New York?
5. The area of Nebraska is 77,237 square miles, of which 584 square miles are inland water. How much of Nebraska is not covered by water?
6. The Erie Canal was completed in 1825. How long ago was that?
7. Going through Memphis, it is 713 miles from New Orleans to St. Louis. It is 307 miles from Memphis to St. Louis. How far is it from New Orleans to Memphis?
8. At Niagara Falls, the American Falls are 167 feet high and 1,400 feet wide. The Canadian Falls are 158 feet high and 2,950 feet wide. Compare the heights and widths of the two parts of Niagara Falls.



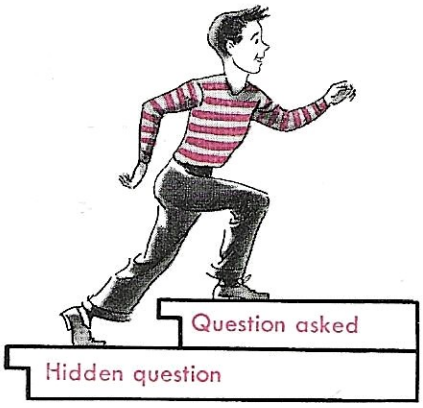
Operations on Whole Numbers

ADDITION AND SUBTRACTION

Write the number of each line of the chart below on a sheet of paper. Do not write in the book. Across the top of the paper, write the headings from the chart. In a line after 1, 2, 3, and each of the other numbers on your paper, write the words or numbers which are missing in the chart. Do not change the order of the numbers operated on. Line 1 is done completely for you to show you how to do the other lines.

Numbers Operated on	Result	Operation Used	Mathematical Sentence
1. 15, 4	11	Subtraction	$15 - 4 = 11$
2. ?	?	Addition	$25 + n = 35$
3. 96, 34	?	Addition	?
4. 60, 34, n	?	?	$60 + 34 + n = 144$
5. 200, 120	?	Subtraction	?
6. 465, n	786	?	?
7. 75, 25, n	135	?	?
8. ?	?	Subtraction	$135 - n = 110$
9. ?	?	Addition	$467 + n = 826$
10. 349, 129	220	?	?
11. 62, n, 40	182	?	?
12. 98, n	38	?	?
13. n, 29, 34	128	?	?
14. 786, 384	?	Subtraction	?
15. 297, n	500	?	?
16. ?	?	?	$734 - 256 = 478$
17. 38, n	27	Subtraction	?
18. 160, 70	?	Addition	?
19. ?	?	?	$47 - 23 = 24$
20. 207, n, 3	13	Subtraction	?

Problems with Two Steps



Sometimes problems have hidden questions; they are like two problems in one.

1. Mrs. Brown bought some cookies for 39¢ and some ice cream for 45¢. She gave the clerk a dollar bill. How much change should she get?

What do you have to know besides what the problem tells you? Before you can find the amount of change, you must add to find how much the two things cost.

```
  39¢
  45¢
 ----
  84¢

$1.00
  .84
 ----
$ .16
```

Now the problem asks how much change Mrs. Brown will receive from a dollar bill if she buys two things that cost 84¢. Subtract to find the answer. How much change should Mrs. Brown get?

There is always more than one correct way to think about a problem. Can you think of a way to do this same problem by using two subtractions? What would the hidden question be if you did the problem in this way? Would you get the same answer?

```
$1.00
  .39
 ----
$ .61

 $.61
  .45
 ----
 $.16
```

Solve the following problems.

2. Joan is reading a book at home that has 350 pages. She read 25 pages on Monday, 60 pages on Tuesday, and 44 pages on Wednesday. How many more pages does she have to read?

3. Tom wants a coat that costs $25.00. For his birthday he received $10.00. During the past month he has saved $6.95. How much more money must he have before he can buy the new coat?

4. Mr. Jones bought a quart of oil for 45¢ and some gas for $2.50. He gave the serviceman a $5.00 bill. How much change did he receive?

5. The sixth grade boys carried 70 small chairs into the auditorium for the first graders. How many extra chairs will there be if one first grade class has 35 pupils and the other has 30 pupils?

What Have You Learned about Addition and Subtraction?

CHAPTER TEST ON MEANINGS AND SKILLS

$$\begin{array}{r} 46 \\ +72 \\ \hline 118 \end{array}$$

$$\begin{array}{r} 784 \\ -432 \\ \hline 352 \end{array}$$

1. In the example at the left, what numbers are the addends? What number is the sum?
2. Use $4 + 8 + 7 + 9 + 2$ to show that changing the order of the addends does not change the sum.
3. In the example at the left, what number is the minuend? the subtrahend? the difference?
4. Which of the following is an eight-place number?
   8 88 52,730,861 8,431,267
5. Which of these numbers means 9 hundreds, 6 tens, 4 ones?
   910061041 469 964 649
6. Write in Arabic numerals: XIV CCC CD MCMLXI

Copy, add, and check.

7. $\begin{array}{rrrrrr} 45 & 57 & 10 & 62 & 718 & 346 \\ \underline{32} & \underline{40} & 40 & 20 & \underline{251} & \underline{233} \\ & & \underline{30} & \underline{15} & & \end{array}$

8. $\begin{array}{rrrrr} \$2.05 & 8765 & 1,497,583 & 25,436 & \$\ 3.54 \\ \underline{.87} & \underline{4245} & \underline{9,745,977} & 79,854 & 72.63 \\ & & & 36,698 & 84.92 \\ & & & \underline{29,959} & \underline{.56} \end{array}$

Copy, subtract, and check.

9. $\begin{array}{rrrrrr} 99 & 78 & 359 & 276 & 157 & 270 \\ \underline{24} & \underline{26} & \underline{105} & \underline{124} & \underline{87} & \underline{160} \end{array}$

10. $\begin{array}{rrrrr} 53 & 426 & 500 & 504,627 & \$124.00 \\ \underline{9} & \underline{279} & \underline{376} & \underline{39,428} & \underline{36.72} \end{array}$

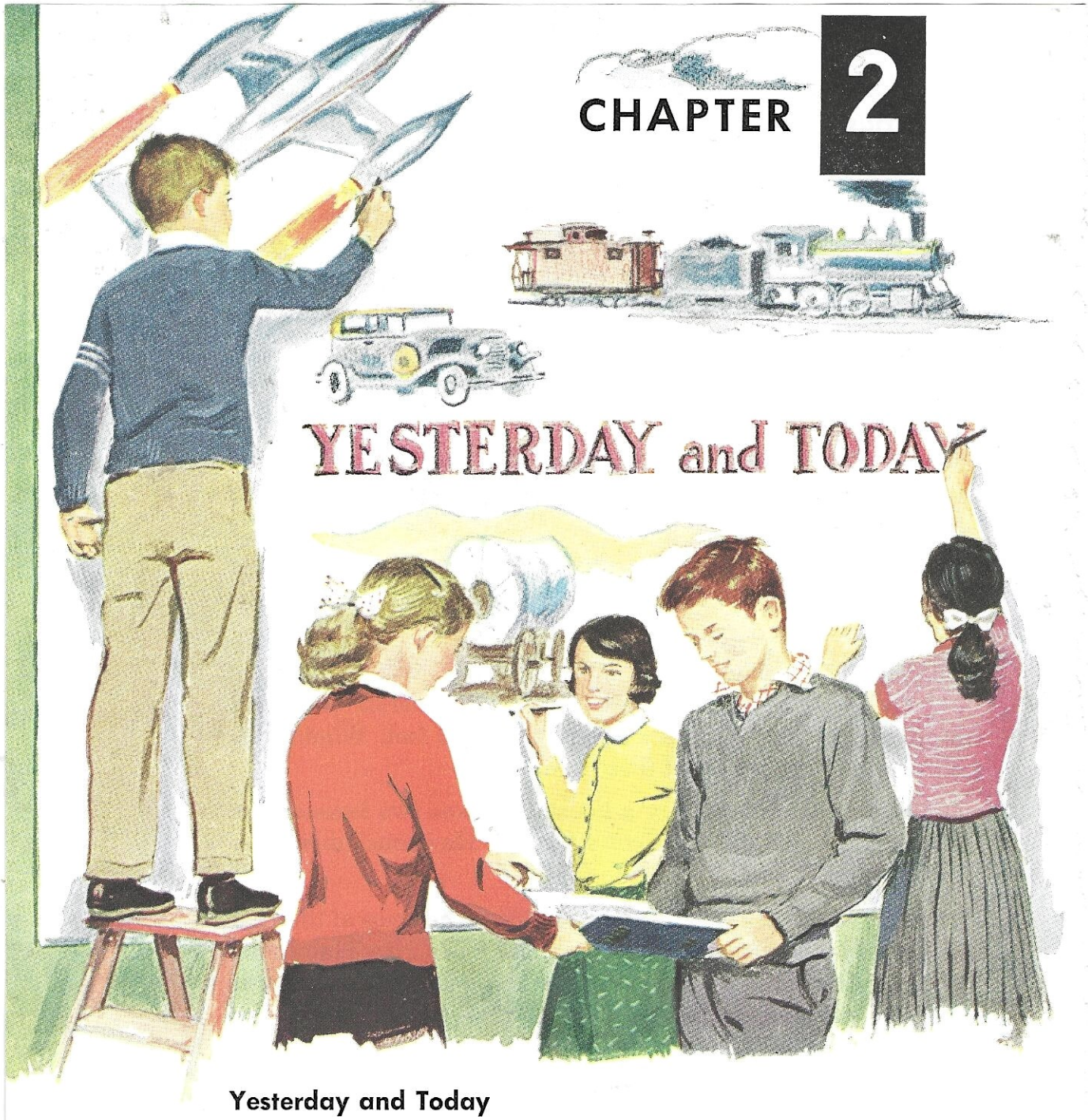
Copy these examples. Do as the signs tell you.

11. $45 + 37 + 21 + 6 + 594$ $\$5.00 - \$2.98$

What Have You Learned about Addition and Subtraction?

CHAPTER TEST ON PROBLEM SOLVING

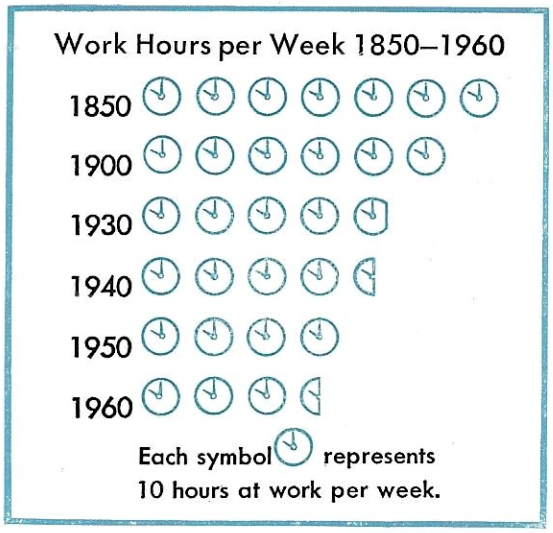
1. Mr. Saunders drove his car from New York City to San Francisco, a distance of 3,030 miles. He sold his car there and returned to New York City by plane, a distance of 2,571 miles. How far did Mr. Saunders travel in all?
2. Mrs. Miller spent $5.34 for groceries, $3.00 for a roast of beef, and $2.68 for hardware. How much money did she spend all together?
3. Jerry's album will hold 248 stamps. He now has 134 stamps pasted in it. How many more stamps does Jerry need to fill his album?
4. A bakery made 9,021 loaves of bread in one day. Their sales slips showed they had sold 8,705 loaves during the day. How many were left?
5. The highest point in Asia is Mount Everest, with a height of 29,141 feet. The highest point in North America is Mount McKinley in Alaska, with a height of 20,300 feet. What is the difference in the height of these mountains?
6. Mike's father plans to buy a house. He likes two houses. One costs $19,600 and the other costs $16,970. How much more does the first house cost?
7. The area of Kansas is 82,276 square miles. The areas of the New England states are: Maine 33,215 square miles, New Hampshire 9,304 square miles, Vermont 9,609 square miles, Massachusetts 8,257 square miles, Rhode Island 1,214 square miles, and Connecticut 5,009 square miles. Is the area of all the New England states put together larger than Kansas?
8. In the election for student council president, 232 boys and 543 girls voted for Gloria and 476 boys and 314 girls voted for John. How many more votes did John receive in this election than Gloria received?



Yesterday and Today

The sixth graders are studying the history of our country. They find that the way people live changes as time goes by. They are wondering what to expect when they are grown up.

Using Whole Numbers in Multiplication and Division



Jack found this pictograph in a newspaper article. It tells how many hours most men worked each week in the years shown.

1. What is the title of the graph? What does it tell?
2. What do the numbers at the left side of the pictograph stand for? The graph was made before 1960. How can 1960 be on the graph?
3. At the bottom of the pictograph you find the statement that each symbol represents _?_. Then $\frac{1}{2}$ symbol represents _?_.
4. Look at the graph for 1950. How can you find the number of hours in a work week for that year? Can you count the symbols by 10's? Can you add $10 + 10 + 10 + 10$? Can you multiply 4×10? Why can you multiply in this case?
5. In 1850, men worked _?_ hours per week. In 1900, _?_ hours.
6. The graph for 1930 shows not quite a whole symbol. Read it: In 1930, men worked almost _?_ hours per week.
7. The graph for 1940 shows a half symbol. $\frac{1}{2}$ of 10 is _?_. In 1940, men worked _?_ hours per week.
8. What does the graph tell you about how the work week has changed in the last hundred years? Did the person who made the graph make a good estimate for 1960?

You add to put numbers together. You can add if the numbers are equal or unequal.
You multiply to put equal numbers together. You can multiply only if the numbers are equal.

Add or Multiply

Carol sometimes helps her father in his store. Should Carol use addition or multiplication to find most quickly what the following customers should pay?

1. Mrs. Brown wanted 8 towels that cost 75¢ each.
2. Mr. Jones wanted a 29¢ pad of paper, a 15¢ box of paper clips, a 5¢ pencil, a 10¢ bottle of ink, and an 8¢ eraser.
3. Mrs. Jack wanted 8 coloring books that cost 25¢ each.

For which examples below would multiplication be the better process to use?

4. $9 + 9 + 9 + 9 + 9 + 9$
5. $8 + 7 + 3 + 12 + 9$
6. $436 + 436 + 436$
7. $24 + 24 + 24 + 24 + 24$
8. $21 + 23 + 25 + 22$
9. $5248 + 5248 + 5248$
10. Work examples 4, 6, 7, and 9 by adding and by multiplying. Did you get the same answer both ways? Should you? In both ways, does the answer tell how many in all? Would you say that multiplying is a short way of adding when the addends are equal?

The numbers in a multiplication example have special names.

The number to be multiplied is called the *multiplicand* → 42
The number to multiply by is called the *multiplier* → $\times 3$
The answer is called the *product* → 126

The sign $\times$ tells us to multiply. The multiplier and the multiplicand are *factors* of the product.

Do You Know the Multiplication Facts?

INVENTORY TEST

You should be able to complete these facts at a glance. Write the products on folded paper.

Make study cards for any facts that you miss. Practice these facts until you can make a perfect score on the test.

1.	2	4	1	3	6	0	1	2	3	5
	2	5	1	1	2	3	9	4	5	0
2.	1	2	8	7	6	5	5	9	0	4
	8	7	3	5	0	8	3	2	4	7
3.	3	4	0	9	1	5	1	7	2	4
	3	8	2	5	4	1	7	0	8	9
4.	2	6	8	4	1	9	2	7	0	5
	9	6	7	6	0	1	1	2	5	6
5.	3	0	4	7	8	9	3	2	9	7
	2	7	1	9	8	6	0	3	9	8
6.	3	8	8	0	5	4	6	8	1	7
	4	5	2	1	7	0	5	4	3	4
7.	1	4	0	8	6	4	9	8	2	7
	2	3	9	6	3	2	7	0	6	1
8.	5	5	7	5	0	6	2	3	5	6
	4	9	7	5	8	4	0	6	2	1
9.	2	9	1	6	1	9	4	3	9	8
	5	0	6	8	5	3	4	9	4	9
10.	3	9	3	6	7	8	0	7	0	6
	8	8	7	9	3	1	0	6	6	7

Visiting a Newspaper Plant

$$\begin{array}{r} 42 \\ \times 3 \\ \hline 126 \end{array}$$

1. Three busloads of children are going to visit a modern newspaper plant. In each bus there are 42 children. How many children are there in all?

Why can you multiply 3×42 to find the answer?

First multiply the ones. $3 \times 2 = \_?\_$. Write 6 in ones place.

Then multiply the tens. $3 \times 4 = \_?\_$. 12 tens = 1 hundred 2 tens. Write 1 in hundreds place and 2 in tens place.

There were 126 children in all.

$$\begin{array}{r} 901 \\ \times 7 \\ \hline 6307 \end{array}$$

2. One of the printing presses uses 901 pounds of paper each hour. How many pounds of paper will it use in 7 hours?

Explain the example at the left. How much is 7×0? How much is any number $\times$ 0?

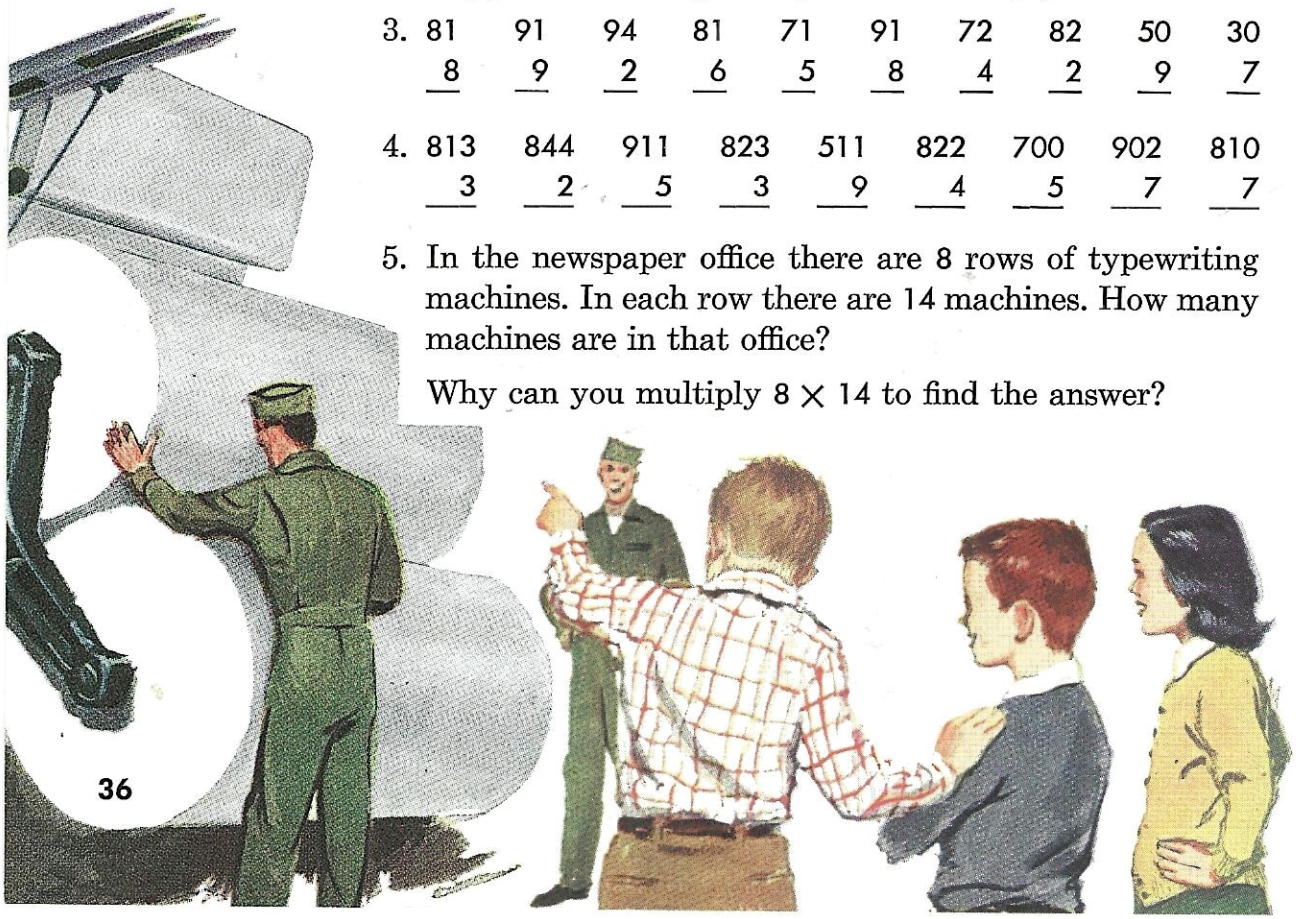
Copy the following examples and multiply.

3. $\begin{array}{rrrrrrrrrr} 81 & 91 & 94 & 81 & 71 & 91 & 72 & 82 & 50 & 30 \\ \underline{\times\ 8} & \underline{\times\ 9} & \underline{\times\ 2} & \underline{\times\ 6} & \underline{\times\ 5} & \underline{\times\ 8} & \underline{\times\ 4} & \underline{\times\ 2} & \underline{\times\ 9} & \underline{\times\ 7} \end{array}$

4. $\begin{array}{rrrrrrrrr} 813 & 844 & 911 & 823 & 511 & 822 & 700 & 902 & 810 \\ \underline{\times\ 3} & \underline{\times\ 2} & \underline{\times\ 5} & \underline{\times\ 3} & \underline{\times\ 9} & \underline{\times\ 4} & \underline{\times\ 5} & \underline{\times\ 7} & \underline{\times\ 7} \end{array}$

5. In the newspaper office there are 8 rows of typewriting machines. In each row there are 14 machines. How many machines are in that office?

Why can you multiply 8×14 to find the answer?



First multiply the ones.

$8 \times 4 =$? 32 ones = ? tens ? ones.

Write 2 in the ones place and keep 3 tens in mind to add with the other tens. This is sometimes called *carrying*.

Then multiply the tens.

$8 \times 1 = 8$ tens. Now add the 3 tens you carried.

$8 \times 1 + 3 =$? 11 tens = ? hundred ? ten.

Write 1 in the hundreds place and 1 in the tens place.

There are 112 machines in that office.

$$\begin{array}{r} 14 \\ \times 8 \\ \hline 112 \end{array}$$

Copy and multiply. Remember to add after multiplying when you have a number to carry.

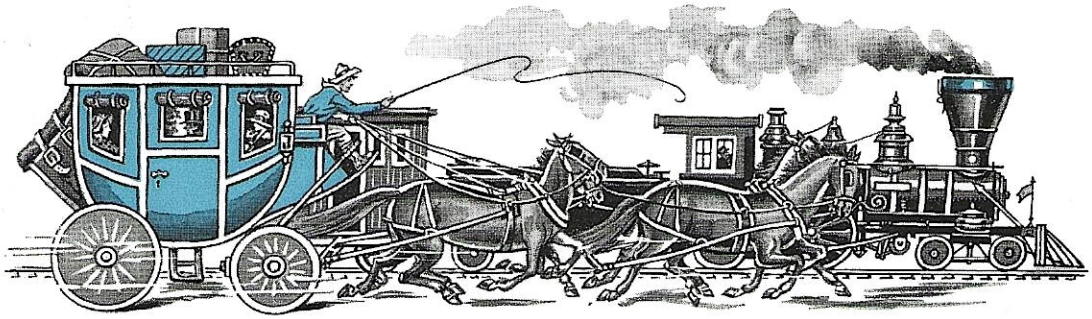
6.	58	87	59	89	72	83	69
	$\times$ 3	$\times$ 6	$\times$ 5	$\times$ 8	$\times$ 9	$\times$ 7	$\times$ 4
7.	919	400	603	307	508	406	6003
	$\times$ 2	$\times$ 9	$\times$ 4	$\times$ 6	$\times$ 8	$\times$ 9	$\times$ 9

Carrying several times in the same example is no harder than carrying once. Study the example at the right. The numbers printed in color are kept in your mind. Think: 9×6 is 54; write 4, carry 5. 9×7 is 63, add the 5, 68; write 8, carry 6. 9×4 is 36, add the 6, 42; write 42. The answer is 4284.

$$\begin{array}{r} {\scriptstyle 65} \\ 476 \\ \times 9 \\ \hline 4284 \end{array}$$

Copy and multiply.

8.	761	872	794	931	973	842	750
	$\times$ 3	$\times$ 4	$\times$ 2	$\times$ 5	$\times$ 2	$\times$ 4	$\times$ 5
9.	765	965	198	496	975	587	478
	$\times$ 8	$\times$ 9	$\times$ 8	$\times$ 7	$\times$ 5	$\times$ 7	$\times$ 9
10.	156	499	562	429	876	695	387
	$\times$ 6	$\times$ 3	$\times$ 8	$\times$ 6	$\times$ 8	$\times$ 4	$\times$ 6
11.	294	867	9000	1200	8090	9006	5104
	$\times$ 9	$\times$ 9	$\times$ 8	$\times$ 9	$\times$ 8	$\times$ 5	$\times$ 8



Yesterday and Today

1. A stagecoach could travel an average of 5 miles an hour. How far could it go in 6 hours?
2. The highest speed that the early locomotives in the United States achieved was 18 miles an hour. How far could these trains go in 8 hours?
3. A modern streamliner train travels 85 miles an hour. How far can it go in 8 hours?
4. In 1895 the speed of an automobile was 8 miles an hour. How far could the car go in 7 hours?
5. Traveling in an automobile at 40 miles an hour, how many miles could you go in 9 hours?
6. In 1947, on Bonneville Flat in Utah, the speed record was broken by a car going 394 miles an hour. How far could this car go in 3 hours? Would a trip like this be possible with today's roads and today's cars and drivers? Do you think it might be possible in the future?
7. During World War I military planes flew 110 miles an hour. How far could they fly in 4 hours?
8. When a plane travels 235 miles in an hour, how far will it travel in 5 hours?
9. The F–84 Thunderjet has a top speed of a little over 600 miles an hour. How far will it fly in 3 hours?
10. A large passenger plane can carry 58 passengers in one trip. How many passengers can it carry in 9 trips?
11. If a helicopter uses 36 gallons of fuel in an hour, how many gallons will it use in 6 hours?
12. The South Station in Boston has an average of 250 trains a day. How many trains does it have in a week?

Working in a Repair Shop

MULTIPLYING NUMBERS OF DOLLARS AND CENTS

1. Ray's older brother plans to work for a radio repair shop 5 hours each day during vacation. He will get 97 cents an hour for his work. How much money will he get for 5 hours?

Why can you say he will get almost $5.00?

To find the exact answer, multiply as you do with any other numbers. When you multiply dollars and cents, remember to write the dollar sign and the decimal point in your answer. Ray's brother will get $4.85. Is that almost $5.00?

Why is there no dollar sign and decimal point with the 5 that is the multiplier in this example?

$.97
×5
$4.85

2. Study these examples. Explain the work. Then copy the examples without the answers. Close your book. Multiply. Compare your answers with those in the book.

$3.25	$4.50	$5.09	$7.00	$60.09
×3	×4	×6	×8	×5
$9.75	$18.00	$30.54	$56.00	$300.45

Copy, multiply, and check by going over your work.

3.	$7.25	$3.50	$6.08	$5.00	$1.89
	2	7	8	9	3
4.	$.38	$.98	$.76	$.75	$.89
	5	6	4	7	9
5.	$7.04	$8.98	$4.89	$3.25	$9.02
	8	5	6	9	9
6.	$60.50	$80.75	$90.05	$65.05	$95.02
	5	6	4	9	8

7. Model auto kits cost $.89 apiece. Carl has 9 of them. How much has he spent for these kits?

Multiplying by Multiples of Ten

Can you multiply a whole number by 10, 100, or 1000 the short way? *To write the product when multiplying a whole number by 10, write a zero after the numeral; by 100, two zeros; by 1000, three zeros.*

Use this rule to give the following products.

1.	10×38	10×52	10×21	10×56
2.	100×36	100×18	100×24	100×57
3.	1000×96	1000×29	1000×34	1000×63

You can multiply by other numbers ending in zero without using a pencil if you think of some things you already know.

You know that 20 is 10×2, so 20 times any number would be 10 times as much as 2 times the number. $2 \times 9 =$ ___?___. Then 20×9 would be $10 \times 18 =$ ___?___.

Use this idea to help you do these examples.

4.	30×2	20×6	40×8	50×9	80×8
5.	40×21	30×23	20×15	60×21	80×51

You know that 400 may be thought of as 100×4, so 400 times any number would be 100 times as much as 4 times the number. $4 \times 3 =$ ___?___. Then 400 times 3 would be $100 \times 12 =$ ___?___.

Use this idea to help you do the following examples.

6.	200×2	200×7	500×5	400×9
7.	500×21	300×82	900×81	800×90

You know that 7000 can be thought of as 1000×7, so 7000 times any number would be 1000 times as much as 7 times the number. $7 \times 2 =$ ___?___. Then 7000×2 would be $1000 \times 14 =$ ___?___.

Use this idea to help you do the following examples.

8.	5000×3	6000×4	8000×5
9.	2000×34	3000×23	7000×90

Two-Place Multipliers

Sixteen airplane engines are to be shipped to a factory. Each engine weighs 812 pounds. How much do all 16 engines weigh?

To find the answer, you could find how much 6 engines weigh, then find how much 10 engines weigh, and add the two answers to find how much 16 engines weigh.

6 engines weigh	10 engines weigh	16 engines weigh
812	812	4872
×6	×10	+8120
4872	8120	12992

1. Explain how the three examples above may be combined into one example, as shown below.

812	multiplicand	812
×16	multiplier	×16
4872	first partial product	4872
8120	second partial product	812
12992	product	12992

2. Look at the first four letters of *partial*. Why are the 4872 and the 8120 called *partial products*?

3. Two ways to write the partial products are shown in example 1. Why can you leave out the zero in the ones place of the second partial product if you remember to put the right-hand figure of this product in the tens place?

Copy and multiply. Check by going over your work.

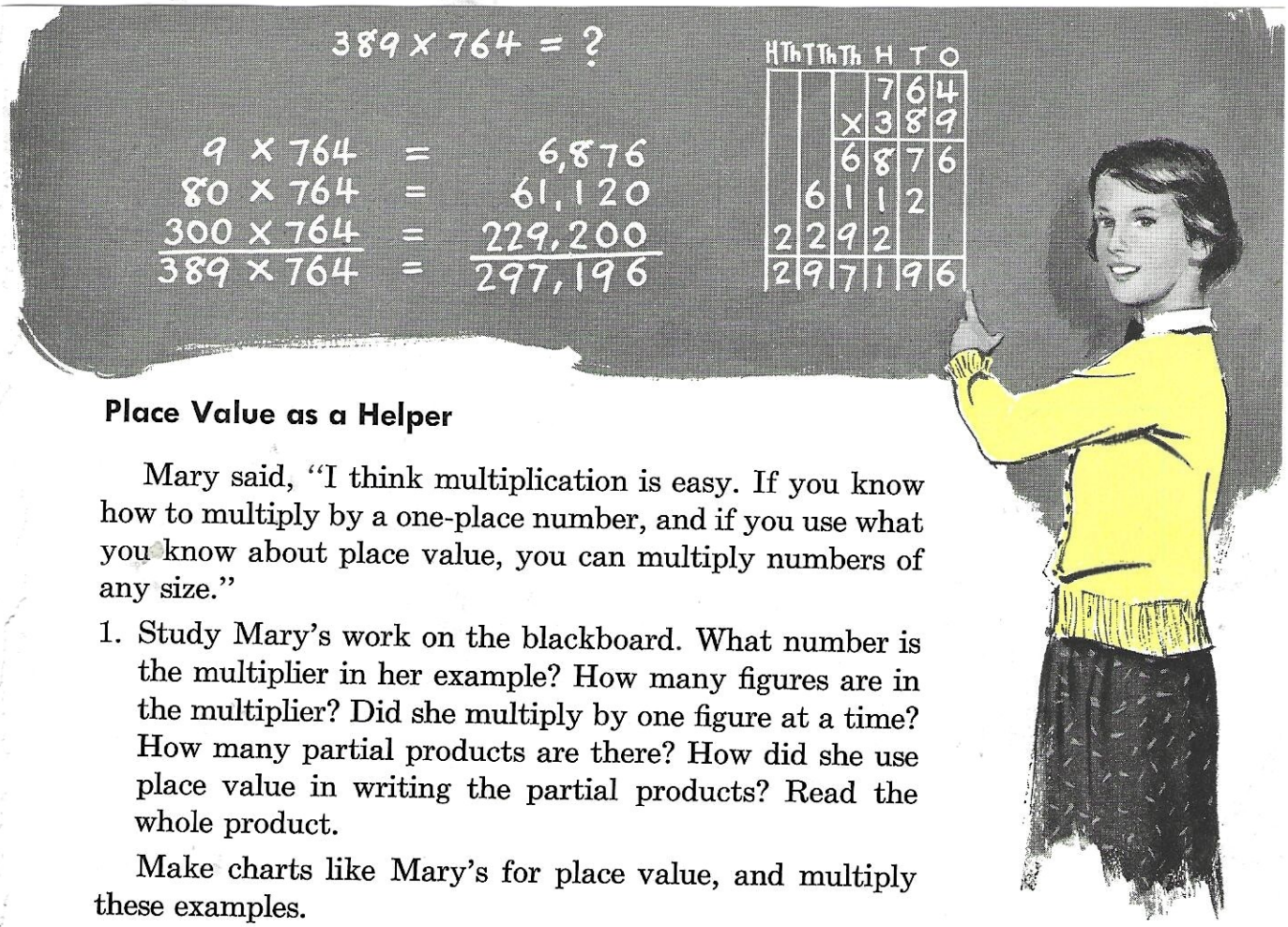
4.	713	915	527	128	211
	15	26	23	47	89
5.	28	67	78	29	63
	43	56	62	87	94
6.	58	92	55	78	69
	25	27	36	48	76

1 nautical mile = 6080 feet

1 land mile = 5280 feet

Two-Figure Multiplier

1. The largest airplane carriers can travel 34 nautical miles an hour. How far can these carriers go in 15 hours?
2. The Yankee clipper ships could travel 300 nautical miles a day. How far could they go in 10 days?
3. The freighter of today travels about 12 nautical miles an hour. How far does it travel in one day? (_?_ hrs. = 1 day)
4. If a vessel burns 260 tons of fuel a day, how much fuel does it burn on a two-week trip?
5. If another vessel burns 316 tons of fuel a day, how much fuel does it burn on a two-week trip?
6. The steamship *United States* can travel 35 nautical miles an hour. How far can it travel in 48 hours?
7. A submarine can travel 20 nautical miles an hour when surfaced. How far can it go in 18 hours?
8. This same submarine can travel only 11 nautical miles an hour under water. At that rate, how far can it go in 18 hours?
9. Each of the 90 submarines that the United States had in 1939 could carry 24 torpedoes. How many torpedoes could all these submarines carry?
10. Some express liners can travel 25 nautical miles an hour. How far can such an express liner travel in 36 hours?
11. A large fishing trawler is manned by 17 men. How many men would be needed to man 25 large trawlers?
12. The *Queen Mary* averages 31 nautical miles an hour. How far can it travel in 48 hours?
13. How much longer is a nautical mile than a land mile?
14. Does the *Queen Mary* travel faster than a car going 31 miles per hour?



Place Value as a Helper

Mary said, "I think multiplication is easy. If you know how to multiply by a one-place number, and if you use what you know about place value, you can multiply numbers of any size."

1. Study Mary's work on the blackboard. What number is the multiplier in her example? How many figures are in the multiplier? Did she multiply by one figure at a time? How many partial products are there? How did she use place value in writing the partial products? Read the whole product.

Make charts like Mary's for place value, and multiply these examples.

2.	846	943	476	6539	3942
	× 732	× 359	× 430	× 63	× 85
3.	5901	6214	3725	4836	5126
	× 678	× 954	× 123	× 879	× 186

Study the two examples at the right. Can 2 and 256 be multiplied either way? Which way is easier? Do both ways give the same answer? Does this give you an idea of a way to check a multiplication example? Changing the position of the numbers like this is called interchanging the multiplier and the multiplicand.

256	2
× 2	× 256
512	12
	10
	4
	512

4. Check your answers to rows 2 and 3 by interchanging the multiplier and the multiplicand in each example.

Zeros in Multiplication

Zero is the easiest number to multiply if you think what you are doing. Zero is not the same as nothing, so you cannot just pretend it isn't there. Sometimes your answer will not be changed if you leave some zeros out of your work, and at those times it makes your work shorter if you do not write the zeros.

John's Work

400
× 70
000
28 00
28,000

Joe's Work

400
× 70
28,000

John and Joe showed two ways to write the work for finding the product of 70 × 400.

John wrote his partial products 0 × 400 = 000 and 7 (tens) × 400 = 2800 (tens). Then he added these partial products.

Joe wrote a zero in the ones place for 0 × 400 and then wrote 2800 at the left of it to show 7 tens × 400 = 2800 tens. Is his answer the same as John's?

A zero may be written in place of the entire partial product of a zero multiplier to show the place value.

Study the following examples. Compare each example with the one below it. Tell why it is not necessary to write all of the partial products in full. Why must you write some of the zeros?

Long Way

300	387	504	327
× 60	× 400	× 120	× 206
000	000	000	1 962
18 00	0 00	10 08	0 00
18,000	154 8	50 4	65 4
	154,800	60,480	67,362

Short Way

300	387	504	327
× 60	× 400	× 120	× 206
18,000	154,800	10 080	1 962
		50 4	65 40
		60,480	67,362

Copy and multiply. Watch place values carefully. Check by interchanging multiplier and multiplicand.

1.	200	400	392	222	333	503
	30	28	400	340	407	707
2.	834	327	435	366	618	206
	106	405	302	407	505	103

Practice with Multiplication

Multiply and check. Remember the dollar sign and the decimal point when your answer means dollars and cents.

1.	64	82	76	901	256	7800
	2	4	8	9	7	6
2.	10×18		20×42			50×72
	70×51		90×21			60×45
3.	74	411	362	76	45	68
	27	72	89	28	45	72
4.	376	224	897	432	115	496
	421	561	567	946	115	387
5.	500	706	403	672	340	400
	60	320	908	600	704	671
6.	\$4.31	\$50.75	\$8.00	\$49.62	\$.18	\$3.50
	7	11	76	16	7	74

Solve the following problems.

7. Before railroads, the Pony Express could average 250 miles a day. At this rate, how many miles were traveled in 5 days?

8. The 1955 Indianapolis 500-mile race was won by Bob Sweikert. His speed was an average of 128 miles per hour. At this rate,

how far could he travel in 4 hours? Use your answer to tell whether his time for the race was more or less than 4 hours.

9. Traveling by airplane at a rate of 325 miles per hour, how many miles could you go in 7 hours?

10. If a car travels 14 miles with one gallon of gasoline, how far will it travel with 16 gallons?

11. Last year Mr. Jones shipped 200 hogs from his stock farm. If the average hog weighed 378 pounds, what was their total weight?

12. During one month a ferryboat made 160 trips with an average of 235 passengers each trip. What was the total number of passengers carried?

13. Louise gets 35¢ a week for doing the supper dishes. How much does she earn from this in a year?

14. Check the following statement:

    If 3 engineers had worked 40 hours a week, 50 weeks every year since Columbus began his voyage in 1492, they still would not have completed the 3,000,000 engineering hours it takes to design and build a large modern airplane.

Just for Fun

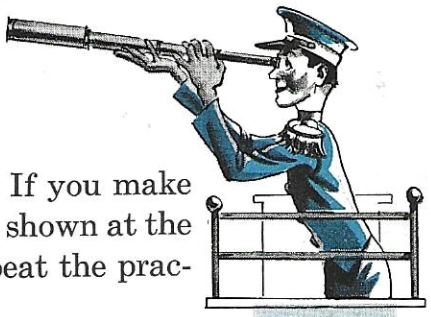
Write the multiplication facts of 9 in a column. Study the column of products. (1) Is it true that the figures in each product add to 9 (18, 1 + 8; 27, 2 + 7; etc.)? (2) Does the tens figure get larger by 1 and the ones figure get smaller by 1 as you read down the column? (3) Could you tell the product of 9 and another number, 9×8, for example, by writing the number next smaller than 8 (7) in tens place in the product, and the difference of $9 - 7$ (2) in ones place? Try the same idea with 9 times some other numbers.

Write the product of 9 and some two-place numbers, such as the numbers from 12 to 21, in a column. See what you can find out about those products.

Helps in Multiplication

Make up several examples that will show how each of the following ideas may help you with the multiplication facts.

1. Changing the order of the numbers you are multiplying does not change the answer. For example, $6 \times 9 = 54$ and $9 \times 6 = 54$. If you know one fact, you know its reverse.
2. Any number $\times$ 1, or 1 $\times$ any number, gives that number.
3. Any number $\times$ 0, or 0 $\times$ any number, gives 0.
4. You can use 10 to help you with 9. For example, $10 \times 7 = 70$, so 9×7 would be 7 less than 70, or 63.
5. If you have other ideas that are useful to you, explain your ideas to the other pupils in your group.



Checking Up on Multiplication

DIAGNOSTIC TEST ON PP. 36–44

Copy the following examples and multiply. If you make mistakes in a row of examples, turn to the page shown at the end of the row. Study the explanation and repeat the practice. Then try the row on this page again.

1.	58 × 4	60 × 3	700 × 9	486 × 8	5006 × 6	36, 37
2.	\$3.25 × 4	\$.98 × 7	\$.57 × 6	\$8.06 × 8	\$70.50 × 5	39
3.	28 × 42	39 × 87	723 × 25	238 × 46	824 × 39	41
4.	832 × 643	467 × 271	503 × 674	4836 × 698	6241 × 375	43
5.	300 × 20	600 × 28	483 × 400	252 × 640	706 × 203	44

Two Reasons for Dividing

35
−7
28
−7
21
−7
14
−7
7
−7
0

5
7)35

1. Arthur has 35 crayons to put in boxes. He can put 7 crayons in each box. How many boxes will he need to put the crayons in?

To find how many boxes he needs, Arthur must find how many groups of 7 crayons there are in 35 crayons.

One way is to subtract 7 again and again until none are left. How many times can he subtract 7? How many groups of 7 are in 35? A shorter way to find how many groups of 7 are in 35 is to divide 35 by 7. $35 \div 7 = \_?\_$. Arthur will use 5 boxes.

Both subtraction and division will give the answer, but division is the shorter way to separate *equal* groups.

One reason for dividing is to find the number of small groups, all the same size, that are contained in a larger group.

In this type of division you know how many there are in each group, and you want to find the number of groups.

2. Miss White had 24 pencils. She gave each of 6 boys the same number of pencils. How many pencils did each boy receive?

You must find how many pencils there would be in each group when 24 pencils are divided into 6 equal groups.

$\frac{1}{6}$ of $24 = 24 \div 6 = \_?\_$ $6\overline{)24}$ Each boy received 4 pencils.

4

Another reason for dividing is to find how many there are in each small group when you separate a larger group into a certain number of smaller groups, all of the same size.

In this type of division, you know the number of groups and you want to find how many there are in each group.

The numbers in a division example have special names.

4 quotient
divisor $6\overline{)24}$ dividend

Do You Know the Division Facts?

INVENTORY TEST

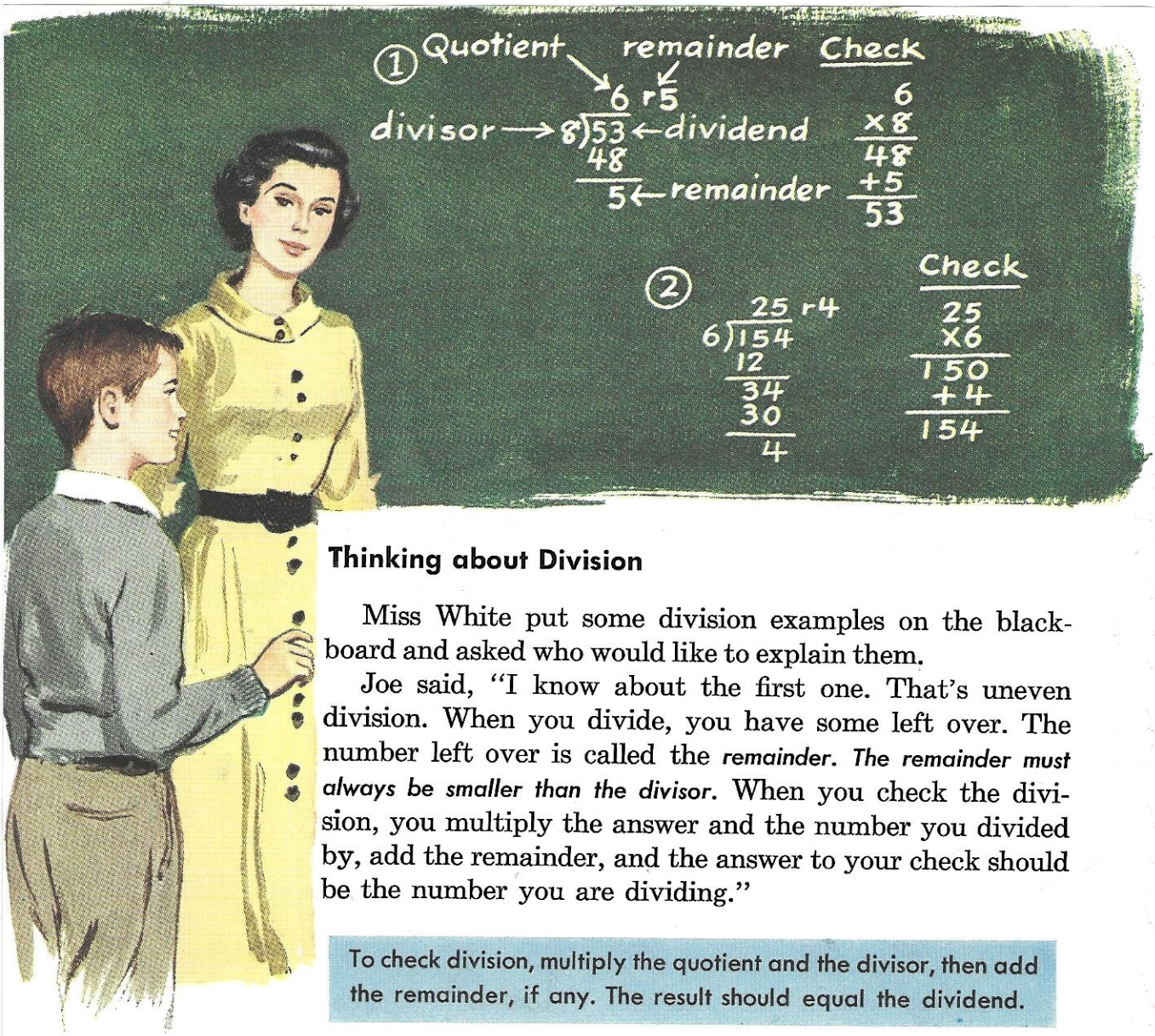
You should be able to complete these facts at a glance. Write the quotients on folded paper.

Make study cards for any facts that you miss. Practice these facts until you can make a perfect score on the test.

1.	$2\overline{)8}$	$4\overline{)12}$	$7\overline{)35}$	$2\overline{)14}$	$7\overline{)63}$	$1\overline{)9}$	$5\overline{)20}$	$6\overline{)54}$	$8\overline{)16}$
2.	$3\overline{)3}$	$4\overline{)28}$	$8\overline{)8}$	$1\overline{)6}$	$6\overline{)42}$	$9\overline{)54}$	$6\overline{)48}$	$8\overline{)64}$	$5\overline{)10}$
3.	$2\overline{)4}$	$6\overline{)6}$	$3\overline{)21}$	$5\overline{)25}$	$3\overline{)24}$	$8\overline{)24}$	$9\overline{)18}$	$8\overline{)72}$	$5\overline{)35}$
4.	$2\overline{)18}$	$9\overline{)45}$	$7\overline{)7}$	$4\overline{)20}$	$5\overline{)15}$	$1\overline{)1}$	$6\overline{)36}$	$3\overline{)12}$	$8\overline{)48}$
5.	$2\overline{)12}$	$6\overline{)24}$	$7\overline{)21}$	$1\overline{)2}$	$4\overline{)32}$	$9\overline{)63}$	$5\overline{)5}$	$2\overline{)6}$	$8\overline{)56}$
6.	$9\overline{)9}$	$7\overline{)42}$	$9\overline{)36}$	$3\overline{)15}$	$1\overline{)5}$	$1\overline{)8}$	$4\overline{)24}$	$6\overline{)18}$	$7\overline{)56}$
7.	$9\overline{)27}$	$6\overline{)12}$	$3\overline{)18}$	$7\overline{)28}$	$7\overline{)49}$	$5\overline{)45}$	$4\overline{)4}$	$3\overline{)27}$	$2\overline{)2}$
8.	$5\overline{)40}$	$1\overline{)7}$	$2\overline{)10}$	$9\overline{)81}$	$4\overline{)8}$	$4\overline{)36}$	$3\overline{)6}$	$1\overline{)4}$	$8\overline{)40}$
9.	$1\overline{)3}$	$4\overline{)16}$	$5\overline{)30}$	$8\overline{)32}$	$2\overline{)16}$	$9\overline{)72}$	$6\overline{)30}$	$3\overline{)9}$	$7\overline{)14}$
10.	$1\overline{)0}$	$2\overline{)0}$	$3\overline{)0}$	$4\overline{)0}$	$5\overline{)0}$	$6\overline{)0}$	$7\overline{)0}$	$8\overline{)0}$	$9\overline{)0}$

Make up several examples that will show how each of the following ideas may help you to remember the division facts.

1. Think of the fact families. If you know $5 \times 9 = 45$, you also know $9 \times 5 = 45$, $45 \div 9 = 5$, and $45 \div 5 = 9$.
2. Any number divided by 1 gives that number; any number divided by itself gives 1.
3. Zero divided by any number gives zero.
4. You cannot divide by zero. (Would it make sense to say, "Divide 6 into 0 equal parts," or "How many 0's in 3"?)



Thinking about Division

Miss White put some division examples on the blackboard and asked who would like to explain them.

Joe said, "I know about the first one. That's uneven division. When you divide, you have some left over. The number left over is called the *remainder. The remainder must always be smaller than the divisor.* When you check the division, you multiply the answer and the number you divided by, add the remainder, and the answer to your check should be the number you are dividing."

To check division, multiply the quotient and the divisor, then add the remainder, if any. The result should equal the dividend.

Copy the following examples, divide, and check.

1.	$5\overline{)49}$	$6\overline{)38}$	$8\overline{)35}$	$7\overline{)55}$	$9\overline{)83}$	$4\overline{)39}$
2.	$7\overline{)50}$	$9\overline{)65}$	$5\overline{)43}$	$4\overline{)34}$	$6\overline{)41}$	$7\overline{)65}$
3.	$8\overline{)71}$	$5\overline{)32}$	$9\overline{)53}$	$3\overline{)40}$	$7\overline{)50}$	$4\overline{)30}$
4.	$7\overline{)57}$	$8\overline{)65}$	$9\overline{)73}$	$7\overline{)58}$	$8\overline{)66}$	$9\overline{)74}$

"I can explain the second example," said Alice. "I can tell that there will be two figures in the quotient because $10 \times 6 = 60$ and $100 \times 6 = 600$ and 154 is between 60 and 600."

How does that tell Alice that there will be two figures in the quotient?

Ken said, "I can estimate the quotient closer than that. I can see that it is not as much as 100, so I multiply $10 \times 6 = 60$, $20 \times 6 = 120$, $30 \times 6 = 180$. The quotient will be between 20 and 30. The first quotient figure will be 2."

How did Ken know that?

Alice said, "I like my way. Since there are two figures in the quotient, the first figure will go in tens place. I find one figure at a time using four steps. Begin with the tens.

Step 1. *Divide.* $15 \div 6 =$?. Write 2 in tens place.

Step 2. *Multiply and compare.* $2 \times 6 =$?. Write 12 under 15. Compare to make sure that 12 is not larger than 15.

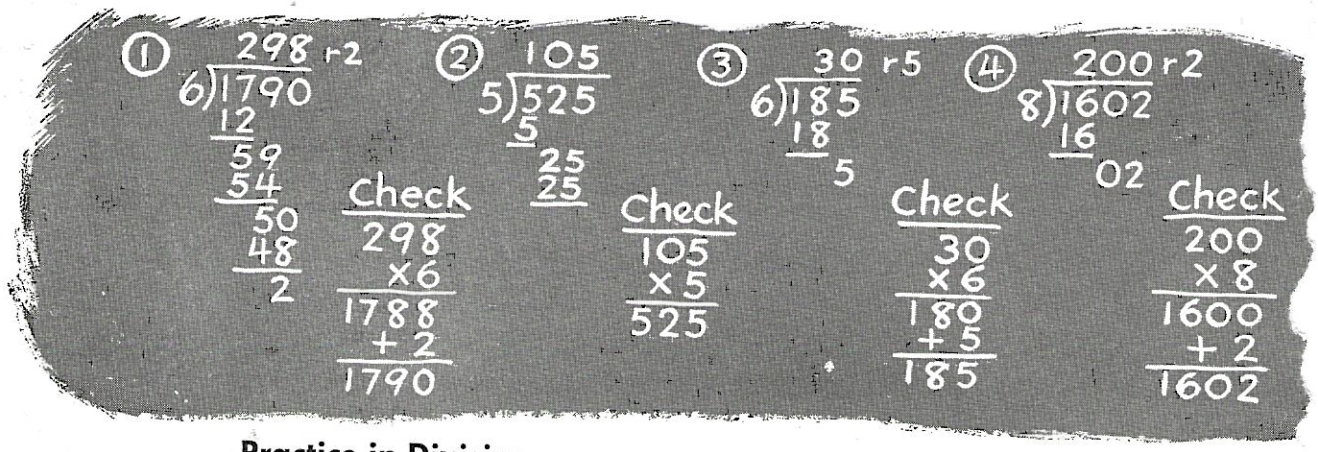
Step 3. *Subtract and compare.* $15 - 12 =$?. Compare the 3 with the divisor to make sure it is smaller than the divisor.

Step 4. *Bring down.* Bring down the 4 ones from the dividend. The 3 tens from the subtraction step and the 4 ones brought down make 34 ones to divide.

"Now use the same steps to divide the ones. The answer is 25 with a remainder of 4."

Be sure that you understand what numbers to compare and how to compare them in Steps 2 and 3. Then copy the following examples, divide, and check.

5.	$2\overline{)157}$	$3\overline{)297}$	$7\overline{)577}$	$5\overline{)197}$	$4\overline{)279}$
6.	$9\overline{)747}$	$8\overline{)672}$	$7\overline{)686}$	$6\overline{)534}$	$5\overline{)378}$
7.	$6\overline{)384}$	$8\overline{)576}$	$4\overline{)392}$	$9\overline{)756}$	$7\overline{)458}$
8.	$7\overline{)593}$	$8\overline{)754}$	$4\overline{)372}$	$5\overline{)295}$	$4\overline{)399}$



Practice in Division

Study the examples on the blackboard. Explain why some of the quotient figures are zeros in the last three examples.

```
  105
5)525
  5
  2
  0
 25
 25
```

Example 2 could have been written as shown at the left. Are there other correct ways to write examples 3 and 4? Notice that after the first quotient figure is written, there must be a figure in the quotient for every figure brought down. Copy the examples without the work. Close your book and do the examples. Then compare your work with the book.

Copy, divide, and check.

1.	$4\overline{)1992}$	$3\overline{)2256}$	$5\overline{)3765}$	$6\overline{)1932}$	$7\overline{)6474}$
2.	$9\overline{)7490}$	$8\overline{)6589}$	$5\overline{)2793}$	$9\overline{)2090}$	$8\overline{)2752}$
3.	$4\overline{)832}$	$3\overline{)924}$	$6\overline{)3624}$	$8\overline{)825}$	$7\overline{)2817}$
4.	$5\overline{)3525}$	$9\overline{)3647}$	$4\overline{)1236}$	$6\overline{)4856}$	$8\overline{)850}$
5.	$7\overline{)4224}$	$5\overline{)3037}$	$9\overline{)5416}$	$4\overline{)1220}$	$6\overline{)3037}$
6.	$8\overline{)243}$	$7\overline{)145}$	$6\overline{)243}$	$5\overline{)204}$	$2\overline{)121}$
7.	$7\overline{)425}$	$4\overline{)202}$	$8\overline{)324}$	$9\overline{)2973}$	$8\overline{)2645}$
8.	$4\overline{)1241}$	$6\overline{)3243}$	$9\overline{)3783}$	$8\overline{)6563}$	$7\overline{)3714}$
9.	$3\overline{)901}$	$4\overline{)803}$	$6\overline{)2405}$	$7\overline{)5602}$	$9\overline{)8108}$

Dividing Numbers of Dollars and Cents

Amounts of dollars and cents may be divided like other numbers except that you have to think what your answer means. If it means dollars and cents, or just cents, it is important to write the dollar sign and decimal point, or a cents sign, in the quotient. If the answer means groups, do not write the money signs.

1. Mary bought 6 pads of paper for \$1.50. How much did one pad cost? Think about the problem: To find out how much each pad cost we have to divide \$1.50 into 6 equal parts. Will the answer mean money? How will we label the answer? In this case, would it be correct to write either 25¢ or \$.25?

$$\begin{array}{r} \$.25 \\ 6\overline{)\$1.50} \\ 12 \\ \hline 30 \\ 30 \\ \hline \end{array}$$

2. How many 5¢ pencils can Jane buy for \$1.25? Think about the problem: To find the answer we have to find how many groups of 5¢ there are in \$1.25. Think of all the money as cents. Pretend that you have 125 pennies you are putting into groups of 5 pennies. Divide 125 by 5. You will have 25 groups of 5 pennies. For every group of 5 pennies, Jane can buy a pencil. How many pencils can she buy?

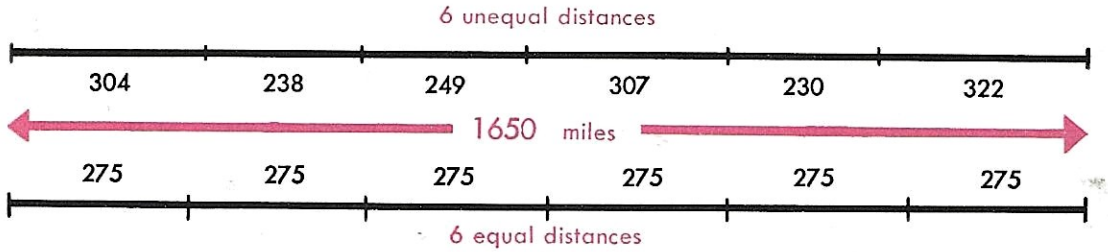
$$\begin{array}{r} 25 \\ 5¢\overline{)125¢} \\ 10 \\ \hline 25 \\ 25 \\ \hline \end{array}$$

3. Ann bought 4 puzzles for 80¢. How much did one puzzle cost?
4. How many 3-cent stamps can Bill buy for 45¢?
5. Joe gets 5¢ a day for making his bed. How many days will it take him to earn \$2.00?
6. If 5 books cost \$2.45, how much will one book cost?
7. If combs cost 4¢ each, how many combs can Sue buy for 40¢?
8. Can you make a rule about how to tell what your answer means when you are dividing money?

Copy the examples and divide. Check your work. Think what each answer means, and label it correctly.

9. $7\overline{)\$6.02}$ $6\overline{)\$3.60}$ $9\overline{)\$36.18}$ $8\overline{)\$50.28}$ $5\overline{)\$28.55}$

10. $2¢\overline{)108¢}$ $4¢\overline{)168¢}$ $7¢\overline{)420¢}$ $8¢\overline{)728¢}$ $9¢\overline{)810¢}$



Finding Averages

Carol's family went on an automobile trip last summer. On Monday they traveled 304 miles; on Tuesday, 238 miles; Wednesday, 249 miles; Thursday, 307 miles; Friday, 230 miles; and Saturday, 322 miles. How many miles per day did they average?

$$\begin{array}{r} 275 \\ 6\overline{)1650} \\ 12 \\ \hline 45 \\ 42 \\ \hline 30 \\ 30 \\ \hline \end{array}$$

Mon.	304 miles
Tues.	238 miles
Wed.	249 miles
Thurs.	307 miles
Fri.	230 miles
Sat.	322 miles
Total for 6 days	1650 miles

To find the average, Carol added the numbers of miles traveled in 6 days and then divided the sum by 6. She knew that her answer should be larger than 230 and smaller than 322. How did she know that?

If the family had traveled the same distance each day, they would have gone 275 miles a day. Their daily average was 275 miles.

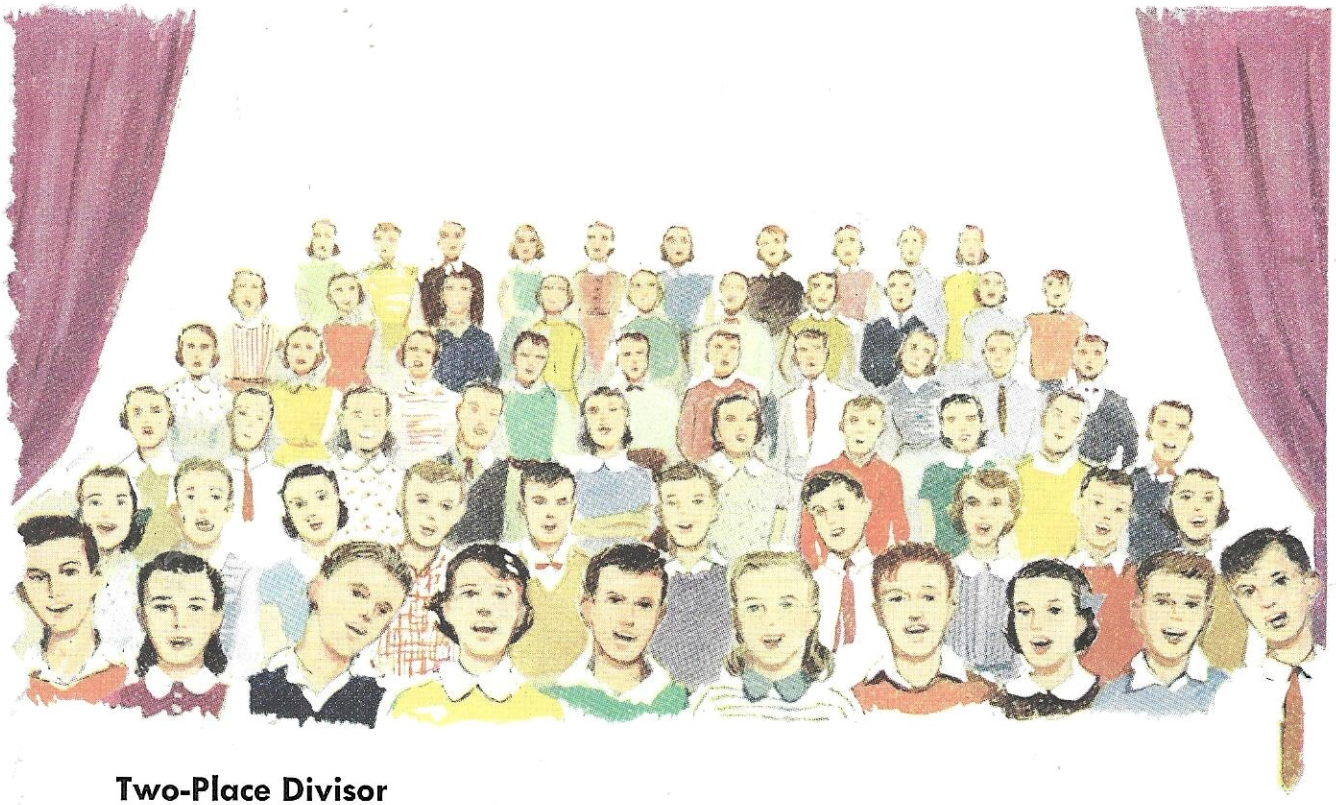
To find an average, first find the sum of the addends and then divide by the number of addends.

1. Find the average weight of the following group: Bob, 92 pounds; John, 86 pounds; William, 83 pounds; and Ralph, 91 pounds.
2. Find the average of the following spelling marks: Monday, 85; Tuesday, 89; Wednesday, 90; Thursday, 76; Friday, 100.

Find the average of each of the following sets of numbers.

3.

56	57	234	$1.75	$2.85
69	49	215	2.15	1.87
47	54	205	3.30	3.13
55	52	238		1.24
60		242		



Two-Place Divisor

1. Sixty pupils are going to sing in assembly. If 10 pupils are to stand in each row, in how many rows will the pupils be placed?

Joe said, "This is a division example. How much is 60 ÷ 10? Let's see. $\underline{?} \times 10 = 60$. $\underline{6} \times 10 = 60$, so there would be 6 rows."

Sue said, "60 ÷ 10 is the same as 6 tens ÷ 1 ten. There are as many 10's in 60 as there are 1's in 6. I can just go by the first figure in each number. I divide just as I would with a one-figure divisor. The quotient is 6."

$$\begin{array}{r} 6 \\ 1\text{ ten}\overline{)6}\text{ tens} \end{array}$$

$$\begin{array}{r} 6 \\ 10\overline{)60} \\ \underline{60} \end{array}$$

Check

$$\begin{array}{r} 10 \\ \underline{6} \\ 60 \end{array}$$

Copy the examples, find the quotient, and check.

2. $10\overline{)80}$ $20\overline{)40}$ $80\overline{)320}$ $70\overline{)420}$ $50\overline{)250}$

3. $30\overline{)150}$ $80\overline{)400}$ $30\overline{)180}$ $40\overline{)120}$ $60\overline{)480}$

Study the examples at the top of the next page.

Be sure you understand how to find the answer to each example below, and how to check it.

```
  6 r5          Check          2 r17          Check          9 r5          Check
10)65            10           30)77            30           20)185           20
 60               6            60               2          180               9
  5              60            17              60            5              180
                  5                            17                           5
                 65                            77                         185
```

Copy, divide, and check.

4. $10\overline{)19}$ $10\overline{)87}$ $30\overline{)122}$ $40\overline{)171}$ $60\overline{)255}$

5. $70\overline{)290}$ $80\overline{)405}$ $50\overline{)275}$ $20\overline{)191}$ $30\overline{)144}$

6. $50\overline{)256}$ $60\overline{)376}$ $30\overline{)136}$ $80\overline{)488}$ $90\overline{)457}$

```
  4 r8
34)144
 136
   8

Check
  34
 ×4
 136
+8
 144
```

7. Janet is making bracelets that require 34 inches of wire. How many bracelets can Janet make from 144 inches of wire?

To find the answer, Janet divided 144 by 34. She said, "I can think of 34 as being about 30. Then there should be about as many 34's in 144 as there are 3's in 14. There are 4 threes in 14."

Try 4 in the quotient. Is it correct? How can you tell?

What does the quotient tell? What does the remainder tell?

Janet can make _?_ bracelets, and she will have _?_ inches of wire left over.

In the check for this example, which numbers stand for a number of inches?

Copy, divide, and check.

8. $75\overline{)600}$ $54\overline{)486}$ $83\overline{)664}$ $63\overline{)315}$ $52\overline{)260}$

9. $72\overline{)648}$ $82\overline{)574}$ $55\overline{)385}$ $44\overline{)352}$ $63\overline{)441}$

10.	$53\overline{)424}$	$72\overline{)432}$	$32\overline{)256}$	$53\overline{)265}$	$32\overline{)288}$
11.	$42\overline{)336}$	$22\overline{)110}$	$43\overline{)258}$	$83\overline{)332}$	$84\overline{)420}$
12.	$65\overline{)325}$	$64\overline{)192}$	$93\overline{)465}$	$33\overline{)132}$	$42\overline{)252}$
13.	$22\overline{)154}$	$42\overline{)294}$	$22\overline{)132}$	$62\overline{)372}$	$52\overline{)312}$
14.	$73\overline{)438}$	$34\overline{)136}$	$35\overline{)105}$	$33\overline{)165}$	$23\overline{)115}$
15.	$32\overline{)195}$	$31\overline{)209}$	$71\overline{)279}$	$61\overline{)530}$	$52\overline{)340}$
16.	$43\overline{)270}$	$55\overline{)449}$	$44\overline{)357}$	$31\overline{)280}$	$81\overline{)630}$
17.	$83\overline{)470}$	$84\overline{)479}$	$85\overline{)311}$	$82\overline{)622}$	$86\overline{)711}$
18.	$75\overline{)552}$	$74\overline{)624}$	$76\overline{)629}$	$76\overline{)481}$	$74\overline{)410}$
19.	$65\overline{)334}$	$55\overline{)394}$	$45\overline{)230}$	$35\overline{)115}$	$36\overline{)110}$
20.	$91\overline{)443}$	$92\overline{)530}$	$93\overline{)628}$	$94\overline{)717}$	$95\overline{)700}$

Solve the following problems.

21. For a craft hobby show Joan is making bracelets from copper wire. Each bracelet needs 32 inches of wire. How many bracelets can Joan make from 128 inches of wire?

22. John is paid 90¢ an hour for painting a neighbor's garage. How long will he have to work to make $4.50?

23. At an average speed of 42 miles per hour, how long will it take to go 336 miles?

24. There are 32 pupils in Mr. Cook's class. They want to buy a large map that costs $2.88. If the pupils share the cost equally, how much will each one pay?

25. A baker uses 48 eggs for one batch of cupcakes. How many dozen eggs is that?

Budgeting Reading Time

TWO-PLACE QUOTIENTS

```
    2
31)651
   62
    3

    2
31)651
   62
   31

   21
31)651
   62
   31
   31

Check
  21
  31
 ---
  21
 63
---
651
```

During December, Mary plans to read two books which have a total of 651 pages. She wants to know how many pages she must plan to read each day to finish the books.

To find the answer, divide 651 by 31. Why use 31?

First, find how many figures there will be in the quotient so you will know where to write the first figure in the quotient. Multiply mentally: $10 \times 31 = 310$; $100 \times 31 = 3100$. The answer will be between 10 and 100 because 651 is more than 310 and less than 3100. The first figure of the quotient will be in the tens place.

Step 1. *Divide* the tens. There are about as many 31's in 65 as there are 3's in 6. Write 2 in the tens place in the quotient.

Step 2. *Multiply and compare.* $2 \times 31 = 62$. 62 is not larger than 65.

Step 3. *Subtract and compare.* $65 - 62 = 3$. 3 is not larger than 31.

Step 4. *Bring down* the 1 one from the dividend.

Go through the same steps with the ones. How can you tell when the division is finished?

Study these divisions and explain each one.

```
  41 r13                22                41 r10
24)997               63)1386            64)2634
  96                  126                256
  37                  126                 74
  24                  126                 64
  13                                      10

Check                Check              Check
  24                   63                 64
 x41                  x22                x41
 ---                  ---                ---
  24                  126                256
  96                  126               2624
 984                 1386               +10
+13                                     ----
---                                     2634
997
```

Copy, divide, and check.

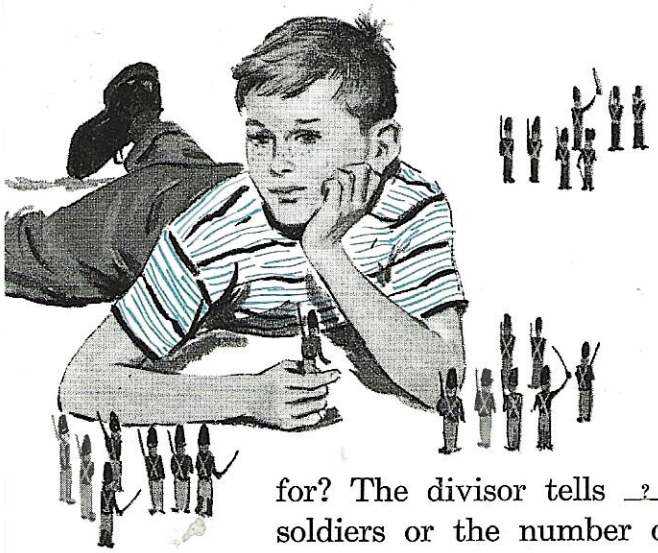
1.	18)198	44)484	51)561	11)561	21)693
2.	11)671	21)483	24)288	23)276	25)275
3.	17)187	92)1288	53)1696	72)1656	54)1188
4.	33)800	35)878	44)706	43)920	54)760
5.	75)987	84)934	45)954	64)838	55)881
6.	35)810	43)919	25)306	24)749	52)1164
7.	72)1596	82)1890	92)1199	81)1140	41)1375
8.	51)1590	71)1582	91)1099	62)1388	34)1410
9.	43)1380	32)1711	42)1728	52)2310	62)2057

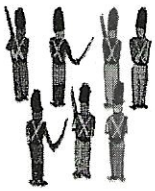
Study the examples at the right, and answer the following questions about each of them: How many figures will there be in the quotient? How do you find the figure in tens place? What do you get when you subtract? After you bring down the ones, how many ones are there to divide? What figure will go in ones place in the quotient? Must you write this zero? Why is it not necessary to write more zeros in the example? How do you check your answer?

```
   40 r8
61)2448
  244
    8

   20 r18
41)838
  82
  18

    80
30)2400
  240
    0
```

Copy, divide, and check.

10.	75)2254	84)2523	63)2521	81)4862	92)2765
11.	33)1658	44)3990	53)3210	45)3160	72)2170
12.	82)6560	54)3800	40)3200	22)1980	68)2040





Thinking about Answers in Division

1. Phil has 29 toy soldiers that he wants to arrange in 4 patrols of the same size. How many soldiers will be in each patrol?

You can see that this is a problem asking you to divide 29 soldiers into 4 equal groups. Write the division example. What does the dividend stand for? The divisor tells _?_. Does the quotient tell a number of soldiers or the number of patrols? If you divide a number of soldiers, does the remainder have to be soldiers? Can Phil have 7 r1 soldiers in each patrol? There are two things that he can do: (1) He can have 4 patrols of 7 soldiers each, and put the remaining soldier aside. (2) He can have 3 patrols with 7 soldiers each, and a fourth patrol with 8 soldiers. Are both ideas sensible? Read the problem again. Since the problem says ". . . 4 patrols of the same size," would the first possibility be the better answer?

2. In Pine Hill School there are 8 sixth grade classrooms. There are 251 children in sixth grade. The principal wants the classes to be about the same size. How many children will be in each class?

The problem really asks you to divide _?_ children into _?_ equal groups. Do the division example. What does the dividend stand for? If you are dividing a number of children, then the remainder must stand for a number of _?_. Can you have 31 r3 children in each class? Can you put the remaining 3 children aside in this case? What would you do with these 3 children if you were the principal? A sensible answer would be, "If the principal makes the classes as nearly the same as possible, there will be 31 children in five of the rooms and 32 children in the other three rooms."

3. How many pairs of socks at $.33 a pair can Jane buy with $2.00?

Think of the money as all cents. You must find how many groups of 33 cents you can make with 200 cents. Do the division example.

Both the dividend and the divisor stand for a number of _?_. If you are dividing a number of cents, then the remainder must stand for a number of _?_. What does the quotient tell in this example? For each group of 33 cents, Jane can buy a pair of socks. A sensible answer to this problem would be, "Jane can buy _?_ pairs of socks. She will have _?_ left over."

Do the check for this example. If you labeled your answer correctly, in the check you will be multiplying a number that means money by a number that tells how many groups there are. When you add the remainder to the product, you will be adding two amounts of money.

$$\begin{array}{r} \$\ .33 \\ \times 6 \\ \hline \$1.98 \\ +.02 \\ \hline \$2.00 \end{array}$$

4. There are 3 children in the Brown family. Their grandmother gave Mr. Brown $5.00 and asked him to give each of the children the same amount. How much should each child get?

The question asks you to divide $5.00 into _?_ equal parts. Do the division example. The divisor is just 3. The dividend tells an amount of money, so the remainder must tell _?_. What does the quotient tell in this example? If you were Mr. Brown, how much money would you give each of the children?

Do the check for this example. Did you multiply a number that means money by a number that tells how many groups there are? Were you adding two amounts of money when you added the remainder?

Solve the following problems. Write each answer in a sentence. Remember that the remainder must always be in the same units as the dividend.

5. How many football teams of 11 players each can you make from a squad of 25 boys?

6. How many 3-cent stamps can you buy with 50 cents?

7. How would you assign 144 children to 5 school buses so that each bus will carry about the same number of children?

8. There will be 20 people at Sally's party. Sally has $4.50 to use for buying favors. How much can she spend for each favor?

Mathematical Sentences

MULTIPLICATION AND DIVISION OF WHOLE NUMBERS

Mathematical sentences can be read when you know the meaning of such symbols as $\times$, $\div$, $=$, $\square$, $\triangle$, n, and others. Symbols such as $\square$, $\triangle$, n, and $\_?\_$ in mathematical sentences represent missing numbers. For example: $5 \times n = 40$, $5 \times \square = 40$, and $5 \times \_?\_ = 40$ ask you to find a missing number. What is the missing number in these sentences?

Copy the following sentences on a piece of paper. Write the numeral that will make each sentence true.

1. $4 \times n = 32$
2. $\triangle \times 22 = 66$
3. $15 \times \square = 300$
4. $n \div 9 = 8$
5. $80 \div \triangle = 20$
6. $\square \times 52 = 364$
7. $n \div 41 = 3$
8. $7 \times \square = 581$
9. $\triangle \times 9 = 81$
10. $144 \div \_?\_ = 12$

When the same symbol is used more than once in the same sentence, it stands for the same number. For example, the boxes in $\square \times \square = 36$ both stand for 6 if the sentence is true. Write this example on your paper as $6 \times 6 = 36$.

Make up some other sentences in which the same symbol is used more than once. Find the numeral which, when substituted for the symbols, will make each sentence true.

On your paper write the numeral that makes each sentence true. Remember the rule.

11. $\square \times \square = 64$
12. $48 \div \triangle = 8$
13. $n \times n = 81$
14. $n \div 9 = 11$
15. $88 \div \_?\_ = 22$
16. $n \times n \times n = 8$
17. $3 \times n = n \times n$
18. $32 \div \square = \square \times 8$
19. $180 \div \triangle = 5 \times \triangle$
20. $n \times n \times n = 64$

Writing the Correct Quotient Figure the First Time

In dividing 120 by 24, two trials are needed to find the correct quotient figure.

You can save time in a division example by finding out if the quotient figure is correct before you write it. To do this you must be able to multiply without writing.

Two sixth graders divided 120 by 24.
One showed both trials on his paper.
The other showed only the second trial. He found that 6 was not the correct quotient figure by multiplying mentally. He thought: "6 × 24 = 144. 144 is larger than 120, so my correct quotient figure must be smaller." He wrote the correct quotient 5 the first time.

First Trial

$$24\overline{)120}\quad 6,\ 144$$

Second Trial

$$24\overline{)120}\quad 5,\ 120$$

Can you multiply 24 by 7 without writing the answer? Think: $7 \times 4 = 28$; $7 \times 20 = 140$; $140 + 28 = 168$. Then $7 \times 24 = 168$. There are other correct ways of multiplying mentally. If you know another way that you like better, use it.

Give these products by multiplying mentally.

1.	3×21	6×91	4×22	5×41
2.	6×36	4×26	9×31	7×21
3.	9×21	3×51	8×35	3×75
4.	6×43	4×32	8×61	6×32

5. Is the product of 6 and 11 more than 66?
6. Is the product of 2 and 66 more than 122?

Copy, divide, and check. Try to write the correct quotient figure the first time.

7.	$75\overline{)215}$	$61\overline{)480}$	$25\overline{)47}$	$63\overline{)305}$	$66\overline{)195}$
8.	$35\overline{)260}$	$32\overline{)185}$	$12\overline{)45}$	$38\overline{)65}$	$41\overline{)360}$

Two-Place Quotient

1. The sixth graders are making Joke Books for the hospital. They decided to make each book 22 pages long. How many books can they make with 826 sheets of paper?

Bill knew that he must divide 826 by 22 to find the answer.

First Quotient Figure

$$\begin{array}{r} 3 \\ 22\overline{)826} \\ 66 \\ \hline 16 \end{array}$$

Second Quotient Figure

$$\begin{array}{r} 37 \text{ r}12 \\ 22\overline{)826} \\ 66 \\ \hline 166 \\ 154 \\ \hline 12 \end{array}$$

First Bill thought: "$10 \times 22 = 220$; $100 \times 22 = 2200$. The quotient will have two figures because 826 is more than 220 but less than 2200. The answer is between 10 and 100."

To find the first figure of the quotient Bill divided 82 by 22. He estimated that the quotient figure would be 4. He multiplied mentally. He found that $4 \times 22 = 88$, so he knew that 4 was too large. He wrote 3 in the tens place of the quotient. He multiplied and compared. 66 is not more than 82. He subtracted and compared. 16 is less than 22.

Then Bill brought down 6 ones from the dividend.

To find the second figure of the quotient, he divided 166 by 22. What figure would he try first? How could he tell 8 was too large? What is the correct quotient figure?

The answer is 37 with a remainder of 12. They can make 37 Joke Books with 12 sheets of paper left over.

$$25\overline{)810}$$

$$\begin{array}{r} 32 \text{ r}10 \\ 25\overline{)810} \\ 75 \\ \hline 60 \\ 50 \\ \hline 10 \end{array}$$

2. Look at the example at the left. How many figures will be in the quotient? How can you tell?

Explain how the first figure of the quotient is found.
Explain how the second figure of the quotient is found.

Copy, divide, and check.

3.	$44\overline{)1649}$	$33\overline{)1875}$	$22\overline{)848}$	$12\overline{)337}$
4.	$53\overline{)1513}$	$42\overline{)1204}$	$15\overline{)485}$	$25\overline{)928}$
5.	$52\overline{)2016}$	$64\overline{)3626}$	$22\overline{)626}$	$63\overline{)2443}$

Correct Quotient Two Less Than Apparent

Sometimes more than two trials are needed to find the correct quotient figure in a division example.

1. Carol is going to the Museum of Science with some of the other children. A group of 196 in all are going by bus. Each bus can carry 28 children. Carol wonders how many buses they will need.

   Carol thought: "196 ÷ 28 is less than 10." Why is this so?

   She estimated 9 as the quotient figure. Why did she choose 9? How could she tell that 9 was too large? Next she tried 8. How did she discover that 8 was too large?

   Then she tried 7. Was 7 the correct quotient?

   How many buses will they need?

$28\overline{)196}$

First Trial

$$\begin{array}{r} 9 \\ 28\overline{)196} \\ 252 \end{array}$$

Second Trial

$$\begin{array}{r} 8 \\ 28\overline{)196} \\ 224 \end{array}$$

Third Trial

$$\begin{array}{r} 7 \\ 28\overline{)196} \\ 196 \end{array}$$

Study Carol's work. It would have saved a lot of work if she had multiplied mentally when trying the different quotient figures.

If you do not like to multiply mentally, you may:

(a) Write each trial separately, as Carol did.

(b) Write the trial figures lightly so that they will be easy to erase if they are incorrect. Erase the trial figures if necessary. When you find the correct quotient figure, write it as usual so that it will be easy to read.

(c) Do the multiplication of the trial quotient figure times the divisor on scrap paper or at the side of your example.

Copy, divide, and check.

2.	$15\overline{)75}$	$14\overline{)98}$	$14\overline{)60}$	$12\overline{)92}$
3.	$35\overline{)2742}$	$24\overline{)1826}$	$25\overline{)\$12.25}$	$24\overline{)\$16.08}$
4.	$24\overline{)1410}$	$23\overline{)1823}$	$25\overline{)1421}$	$35\overline{)\$24.15}$
5.	$38\overline{)\$18.65}$	$25\overline{)\$14.25}$	$28\overline{)\$15.64}$	$24\overline{)\$16.32}$

Ideas to Use in Division

1. A single quotient figure is never more than 9.
2. If a two-place divisor ends in 6, 7, 8, or 9, it usually saves time to use 1 more than the original tens figure in estimating the quotient. Think of 67 as 70, think of 26 as 30, think of 49 as 50, and so on.
3. Be sure to compare after multiplying to make sure that your estimated quotient figure is not too large. Be sure to compare after subtracting to make sure your estimated quotient figure is not too small.

Miss White said, "Let's see if we can work out some ideas to keep us from making mistakes and to help us get the correct answers more quickly."

She asked Jim to divide 217 by 24. Jim said, "10 × 24 = 240. 217 is less than 240, so I will have a one-figure quotient. There are about as many 24's in 217 as there are 2's in 21, or 10. That can't be right, because the largest figure you can have in any place is 9. I'll try 9."

```
    9 r1
24)217
   216
     1
Check
    24
   ×9
   216
  +1
   217
```

1. Is 9 the correct quotient? If 9 had been too large, what would Jim have done next?

Copy and divide. Check your answers.

2. 45)408 84)623 33)306 42)411 46)415

3. 24)226 53)507 44)4110 63)6356 56)6115

Miss White asked if anyone could divide 2000 by 48 and get the correct quotient figure in one trial.

```
   41 r32
48)2000
  192
   80
   48
   32
```

Carl said, "I know an idea that usually works. 48 is nearer to 50 than to 40, so you can use 5 to divide by instead of 4 in estimating the quotient figures."

4. Carl knew that there would be two figures in the quotient. How did he know that? He thought: "There are about as many 48's in 200 as there are 5's in 20, or 4." Is 4 correct for the first quotient figure? How can you tell?

To estimate the second quotient figure, Carl divided 8 by 5. Why did he divide by 5? Is 1 the correct quotient figure? Check the answer.

Copy, divide, and check.

5. $38\overline{)2400}$ $56\overline{)3114}$ $37\overline{)1609}$ $88\overline{)2091}$ $66\overline{)3595}$

6. $59\overline{)133}$ $57\overline{)1356}$ $86\overline{)3858}$ $27\overline{)1830}$ $39\overline{)2050}$

7. $77\overline{)3260}$ $59\overline{)1394}$ $69\overline{)1595}$ $48\overline{)3519}$ $37\overline{)1607}$

Then Miss White said, "Carl increased the tens figure of the divisor by 1 in estimating his quotient figures. That idea usually saves time when the second figure of the divisor is 6, 7, 8, or 9. But it does not always give the correct quotient figure. Sometimes the quotient figure will be too small. Try dividing 140 by 27, using Carl's idea."

Alice said, "This will be a one-place quotient. 27 is closer to 30 than to 20, so I'll use $14 \div 3$ to estimate the quotient."

8. She wrote 4 in the quotient, multiplied, and compared. 108 is not more than 140, so she subtracted and compared. 32 is more than 27. There are more than four 27's in 140. Next Alice tried 5. Is 5 the correct quotient figure? How can you tell? What is the largest remainder you can have when dividing by 27? By any number?

Copy, divide, and check. Be sure to compare each time after you multiply and after you subtract.

9. $28\overline{)168}$ $36\overline{)185}$ $49\overline{)299}$ $48\overline{)340}$ $58\overline{)469}$

10. $38\overline{)798}$ $27\overline{)859}$ $29\overline{)1189}$ $67\overline{)3458}$ $48\overline{)2994}$

11. $17\overline{)101}$ $15\overline{)104}$ $16\overline{)1000}$ $14\overline{)960}$ $13\overline{)1030}$

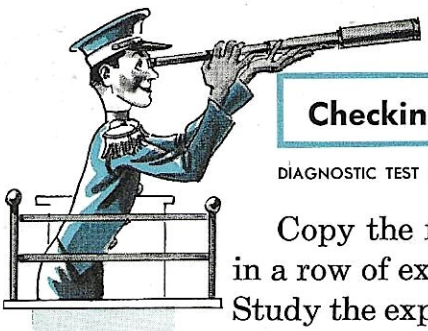
12. $19\overline{)675}$ $18\overline{)1400}$ $15\overline{)645}$ $16\overline{)560}$ $19\overline{)580}$

$$\begin{array}{r} 4 \\ 27\overline{)140} \\ 108 \\ \hline 32 \end{array}$$

$$\begin{array}{r} 5 \text{ r}5 \\ 27\overline{)140} \\ 135 \\ \hline 5 \end{array}$$

Check

$$\begin{array}{r} 27 \\ \times 5 \\ \hline 135 \\ +5 \\ \hline 140 \end{array}$$



Checking Up on Division

DIAGNOSTIC TEST ON PP. 50–65

Copy the following examples and divide. If you make mistakes in a row of examples, turn to the page shown at the left of the row. Study the explanation and repeat the practice. Then try the row on this page again.

50–51	1. $7\overline{)52}$	$8\overline{)49}$	$6\overline{)57}$	$9\overline{)657}$	$5\overline{)483}$
52	2. $3\overline{)1956}$	$6\overline{)2523}$	$7\overline{)3514}$	$8\overline{)8496}$	$9\overline{)2705}$
53	3. $7\overline{)\$9.02}$	$6\overline{)\$4.20}$	$9\overline{)\$45.18}$	$5¢\overline{)275¢}$	$4¢\overline{)248¢}$
56	4. $10\overline{)87}$	$30\overline{)132}$	$50\overline{)285}$	$80\overline{)418}$	$90\overline{)554}$
56	5. $82\overline{)575}$	$44\overline{)342}$	$34\overline{)170}$	$42\overline{)299}$	$94\overline{)707}$
58	6. $45\overline{)945}$	$34\overline{)782}$	$92\overline{)1210}$	$52\overline{)2411}$	$85\overline{)4265}$
64	7. $21\overline{)814}$	$33\overline{)927}$	$53\overline{)1522}$	$45\overline{)1275}$	$74\overline{)3524}$
65	8. $25\overline{)164}$	$23\overline{)1806}$	$35\overline{)\$27.25}$	$25\overline{)\$14.20}$	$24\overline{)\$16.00}$

Practice to Remember

1. Write in figures: Nine million, two hundred fifty-six thousand, seventy-four.
2. Write in words: 6,032,819.
3. Write in Arabic numbers: CX; XLIV; MCM.
4. Write in a column the words *addend*, *sum*, *subtrahend*, *minuend*, and *difference*. In the examples at the right find a number that fits each word and write it beside the word.

$$\begin{array}{r} 78 \\ +601 \\ \hline 679 \end{array} \qquad \begin{array}{r} 432 \\ -197 \\ \hline 235 \end{array}$$

5. Write the fact family to which $9 + 8 = 17$ belongs.

Copy. Add down. Check by adding up.

6.

216	582	56,842	$.42	$541.92
352	495	37,213	.56	5.17
	579	40,506	.07	21.04
		71,254	.21	.07
			.56	518.40

Copy. Subtract. Check by adding.

7.

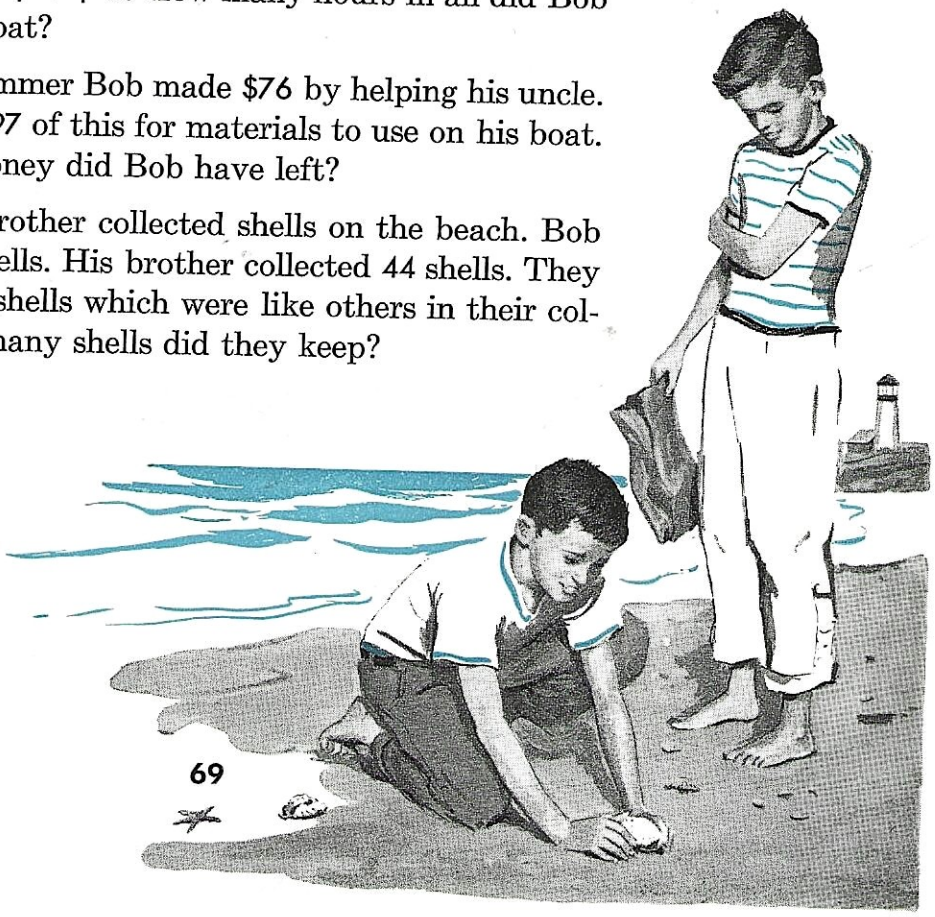
62	236	400	56,721	$34,756.42
3	127	231	34,792	5,492.03

Solve the following problems.

8. Bob spent the last six days of his vacation getting his boat ready for the winter. He kept a record of the number of hours he worked each day. His record read $7 + 2 + 8 + 0 + 4 + 5$. How many hours in all did Bob work on his boat?

9. During the summer Bob made $76 by helping his uncle. He spent $15.97 of this for materials to use on his boat. How much money did Bob have left?

10. Bob and his brother collected shells on the beach. Bob collected 46 shells. His brother collected 44 shells. They gave away 19 shells which were like others in their collection. How many shells did they keep?



Operations on Whole Numbers

MULTIPLICATION AND DIVISION

Write the number of each line of the chart below on a sheet of paper. Do not write in the book. Across the top of the paper, write the headings from the chart. In a line after 1, 2, 3, and each of the other numbers on your paper, write the numbers which are missing in the chart. Line 1 is done completely for you to show you how to do the other lines.

Mathematical Sentence	Numbers Operated on	Operation Used	Missing Number
1. $6 \times 8 = n$	6, 8	$\times$	48
2. $n \times 9 = 63$	9, 63	$\times$ (or $\div$)	?
3. $21 \div 7 =$?	21, 7	$\div$	?
4. $72 \div 8 = n$	72, 8	$\div$	?
5. $5 \times n = 50$	50, 5	$\times$ (or $\div$)	?
6. $n = 9 \times 21$	9, 21	$\times$	?
7. $n = 357 \div 7$	357, 7	$\div$	?
8. $11 \times 32 = n$	11, 32	$\times$	?
9. $n \times 44 = 484$	44, 484	$\times$ (or $\div$)	?
10. $484 \div 22 = n$	484, 22	$\div$	?
11. ?	56, 6	$\times$	?
12. ?	8, 9	$\times$	?
13. $71 \times 10 = n$	71, 10	$\times$	?
14. ?	80, 9	$\times$	?
15. $756 \div 9 = n$	756, 9	$\div$	?
16. $n = 15 \times 13$	15, 13	$\times$	?
17. $n = 102 \div 17$	102, 17	$\div$	?
18. ?	12, 33	$\times$	?
19. ?	7, 8	$\times$	?
20. $41 \times 4 = n$	41, 4	$\times$	?

Problems with Two Steps

1. Mr. Brown is going to take 11-year-old Jerry to the circus. Children under 12 years go for half price. If Mr. Brown buys $3.00 tickets, how much will the two tickets cost?

2. Jack earns 75¢ an hour. Saturday he earned $2.25 in the morning and $1.50 in the afternoon. How many hours did he work?

3. A music store advertises long-playing records at $1.89 each, or 6 for $10.98. By buying 6 records, how much less would you pay per record?

4. Miss White's class is making scrapbooks. Each of 17 children will need 18 sheets of colored drawing paper. The rest of the class will need 304 sheets. How many sheets will the class need all together?

5. Miss Jones's class is making 32 kites for the school fair. The materials for each kite cost $.17. If all the kites are sold at $.25 each, what will the profit be?

6. Dan and Joe are in charge of printing the class newspaper. For one edition they print 32 copies of 2 sheets each. How many editions can they print from a package of 500 sheets of paper?

7. The "Starfighter," a jet plane, can fly 1400 miles an hour. The air distance between New York and California is about 3000 miles. How many miles more is this than a 2-hour flight of the "Starfighter"?

8. Last year the Smiths drove 1789 miles on their vacation. This year they plan to drive from Nashville to Atlantic City, 864 mi.; from Atlantic City to Boston, 335 mi.; from Boston to New York, 217 mi.; from New York to Nashville, 915 mi. How many miles farther do they plan to drive this year than last?

What Have You Learned about Multiplication and Division?

CHAPTER TEST ON MEANINGS AND SKILLS

1. In the example on the right, what number is the product? The multiplicand? The multiplier?

$$\begin{array}{r} 476 \\ \times 3 \\ \hline 1428 \end{array}$$

$$8\overline{)465}\ \text{r}1$$

2. In the example on the left, what number is the divisor? The remainder? The dividend? The quotient?

3. Use two ways to find the total of 6, 6, 6, and 6.

4. Joan said, (a) "I have divided 27 pennies equally among my 3 brothers. Each one received $\frac{1}{3}$ of 27 pennies, or _?_ pennies." (b) "I have 27 pennies to divide into groups of 9 each. There will be _?_ groups of pennies."

   Which problem shows that when you know how many are in each group, you divide to find the number of groups?

   Which problem shows that when you know how many groups there are, you divide to find how many in each group?

Copy and multiply. Check by going over your work.

5.	61 × 6	633 × 2	540 × 6	\$34.07 × 9	\$.37 × 8	\$47.65 × 7
6.	65 × 20	69 × 71	437 × 14	580 × 96	405 × 96	47 × 100
7.	700 × 463	726 × 547	806 × 309	768 × 500	480 × 270	444 × 978

Copy, divide, and check.

8.	$8\overline{)58}$	$9\overline{)643}$	$8\overline{)3020}$	$7\overline{)400}$	$3\overline{)422}$	$9\overline{)994}$
9.	$3¢\overline{)282¢}$	$8\overline{)\$40.32}$	$80\overline{)\$7.65}$	$40\overline{)\$3.71}$	$52¢\overline{)126¢}$	$\$.44\overline{)\$1.87}$
10.	$27\overline{)197}$	$93\overline{)600}$	$63\overline{)3614}$	$78\overline{)3524}$	$38\overline{)2407}$	$26\overline{)2536}$

What Have You Learned about Multiplication and Division?

CHAPTER TEST ON PROBLEM SOLVING

1. How many miles can an automobile travel in 7 hours if its average speed is 46 miles an hour?
2. At 450 miles an hour, how far can an airplane fly in 8 hours?
3. A travel agency advertises an eight-day trip from New York to Washington for $152.50. What would it cost for 9 people to take this trip?
4. A modern train maintains an average speed of 56 miles an hour. How many miles does it travel in 36 hours?
5. When starting the Pony Express, the officials bought 420 horses at $200 each. How much did they spend for horses?
6. In making the runs for the Pony Express, the riders changed horses about every 12 miles. How many horses would be used in a run of 156 miles?
7. In one of the fastest runs, 285 miles were covered in 15 hours. What was the average speed?
8. The Pony Express ran 1966 miles from St. Joseph, Missouri to Sacramento, California. If each rider traveled 75 miles, how many riders would be needed to travel 1950 miles? If some riders traveled a little more than 75 miles, could the same number of men ride the whole distance?
9. If 1 ticket to a movie about the Pony Express costs 32 cents, how many tickets can you buy with $3.00? What does the remainder tell in this example?
10. The 56 boys at Day Camp went for a ride on a lake steamer. After the tickets were bought, there was $16.80 left in the treasury. This was divided equally among the boys for spending money. How much did each boy have to spend?
11. At $1.54 each, how much would 56 tickets cost?

CHAPTER 3

Fractions Are Parts

1. Find the cake that is cut into two equal parts. What is one part of the cake called? What are both parts called?

A fraction may mean one or more equal parts of a single thing.

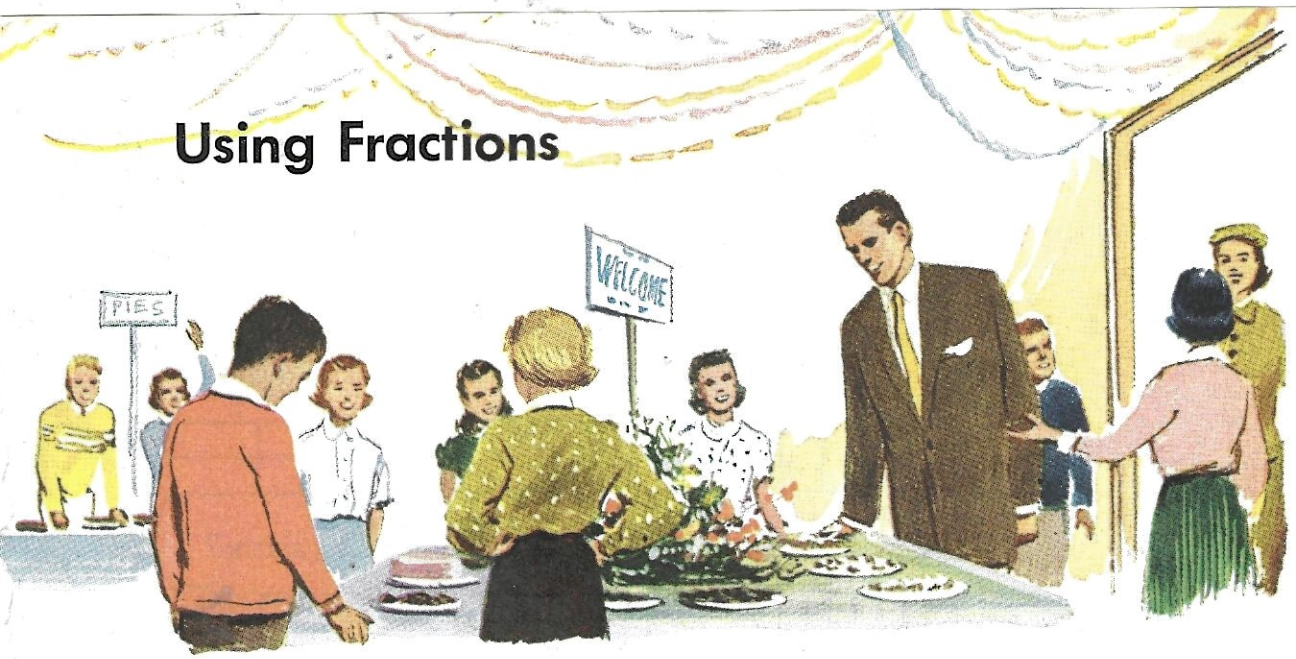
2. Find the pan of nine rolls. Each roll is what fraction of all the rolls? Five rolls are what fraction of all the rolls?

A fraction may mean one or more equal parts of a group.

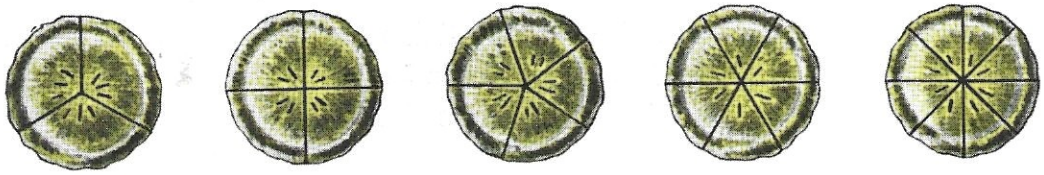
3. Find the large pie that has been cut into nine equal pieces. Each piece is what fraction of the whole pie? Five pieces are what fraction of the whole pie?

4. How are the fractions about the rolls and the pie alike? How are they different?

Using Fractions



5. For each pie in the picture below, write the fraction that tells what part of the whole pie one piece would be. How many numbers did you have to use in writing each fraction?



A fraction has two *terms,* a *numerator* and a *denominator.* The *numerator* tells the number of parts you are speaking of. The *denominator* tells into how many equal parts the whole has been divided. In this way, it tells the size of the parts.

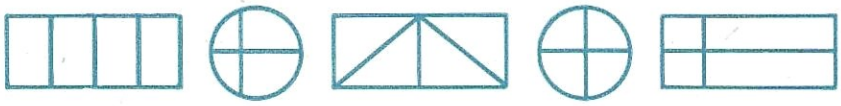
numerator / denominator	$\frac{1}{6}$	means	1 of the 6 equal parts	of the whole

$\frac{1}{6}$ may also be written *1 sixth* or *one sixth.*

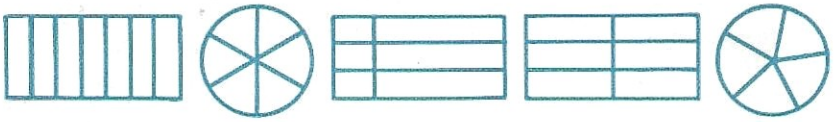
6. For each pie in the picture above, write the fraction that tells what part of the whole two pieces would be. What does each of these fractions mean?

7. Write in figures and in words the fraction shown by the shaded part of each circle below.

8. Which of the following are divided into fourths?



9. Which of the following are divided into sixths?

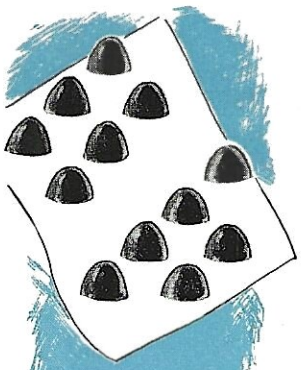


10. Write a sentence about four parts and fourths. About six parts and sixths. What word must you use in each of your sentences?

11. Is one fourth of a small circle the same size as one fourth of a large circle? Are all fourths the same size? Are all fourths of the same circle the same size? Are all fourths of equal circles the same size?

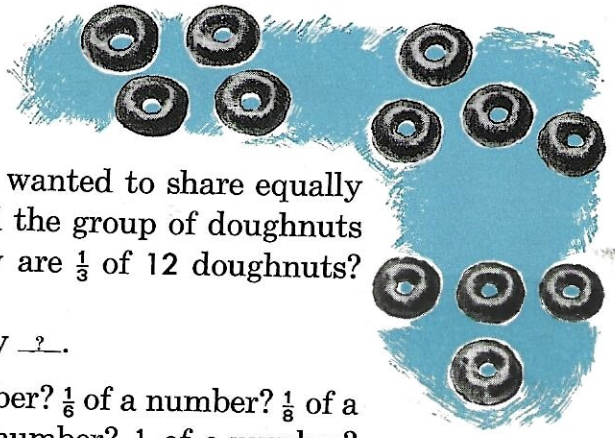
Finding a Part of a Number

UNIT FRACTION OF A NUMBER



1. Jerry brought 12 pieces of candy to school. He wanted to give Bob $\frac{1}{2}$ of the candy. How many are $\frac{1}{2}$ of 12 pieces of candy? To find $\frac{1}{2}$ of 12, Jerry divided 12 into 2 equal parts.
   $12 \div 2 = 6$, so $\frac{1}{2}$ of $12 = 6$.
   To find $\frac{1}{2}$ of a number, divide by _?_.

2. Joan wanted to share equally with her 3 friends the 12 cookies her mother gave her. She divided the group of cookies into 4 equal parts. How do you find $\frac{1}{4}$ of 12?
   $12 \div 4 = 3$, so $\frac{1}{4}$ of $12 = 3$.
   To find $\frac{1}{4}$ of a number, divide by _?_.



3. Bill had 12 doughnuts that he wanted to share equally with Joe and John. He divided the group of doughnuts into 3 equal parts. How many are $\frac{1}{3}$ of 12 doughnuts?
   $12 \div 3 =$?, so $\frac{1}{3}$ of $12 =$?.
   To find $\frac{1}{3}$ of a number, divide by ?.

4. How would you find $\frac{1}{5}$ of a number? $\frac{1}{6}$ of a number? $\frac{1}{8}$ of a number? $\frac{1}{7}$ of a number? $\frac{1}{9}$ of a number? $\frac{1}{10}$ of a number? $\frac{1}{93}$ of a number?

How many are:

5. $\frac{1}{5}$ of 10 pencils? $\frac{1}{8}$ of 16 rolls?

6. $\frac{1}{6}$ of 12 cupcakes? $\frac{1}{7}$ of 14 candy bars?

7. $\frac{1}{2}$ of 10 crayons? $\frac{1}{3}$ of 9 balls?

8. $\frac{1}{2}$ of 22? $\frac{1}{5}$ of 15? $\frac{1}{6}$ of 18? $\frac{1}{10}$ of 40?

9. $\frac{1}{3}$ of 27? $\frac{1}{9}$ of 18? $\frac{1}{5}$ of 20? $\frac{1}{7}$ of 63?

10. $\frac{1}{4}$ of 28? $\frac{1}{2}$ of 100? $\frac{1}{2}$ of 486? $\frac{1}{3}$ of 276?

11. $\frac{1}{4}$ of 168? $\frac{1}{3}$ of 129? $\frac{1}{5}$ of 500? $\frac{1}{8}$ of 168?

12. $\frac{1}{2} \times 6 = \frac{1}{2}$ of $6 =$? $\frac{1}{3} \times 6 =$? $=$?

    $\frac{1}{2} \times 8 =$? $=$? $\frac{1}{4} \times 12 =$? $=$?

    $\frac{1}{5} \times 15 =$? $=$? $\frac{1}{8} \times 16 =$? $=$?

Find:

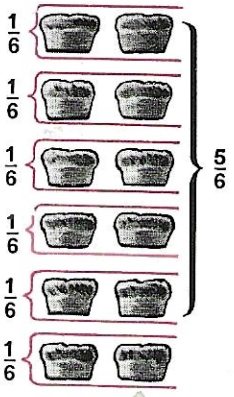
13. $\frac{1}{2} \times 10$ $\frac{1}{4} \times 48$ $\frac{1}{9} \times 81$ $\frac{1}{10} \times 100$

14. $\frac{1}{3} \times 21$ $\frac{1}{9} \times 72$ $\frac{1}{6} \times 42$ $\frac{1}{8} \times 648$

15. $\frac{1}{8} \times 24$ $\frac{1}{7} \times 56$ $\frac{1}{5} \times 40$ $\frac{1}{7} \times 427$

Finding a Part of a Number

MULTIPLE FRACTIONS OF A NUMBER



1. Mother put 12 rolls on the table for supper. After supper Jane said, "We ate $\frac{5}{6}$ of the rolls." How many rolls did they eat?

   How do you find $\frac{1}{6}$ of 12? $\frac{5}{6}$ of 12 would be _?_ times as much as $\frac{1}{6}$ of 12. $\frac{1}{6}$ of $12 = 2$. $\frac{5}{6}$ of $12 = 5 \times 2 = 10$. Jane's family ate _?_ rolls.

2. You know that $\frac{1}{5}$ of $15 = 3$, so $\frac{2}{5}$ of $15 = 2 \times$ _?_ = _?_.

3. You know that $\frac{1}{8}$ of $40 = 5$, so $\frac{7}{8}$ of $40 =$ _?_ $\times 5 =$ _?_.

How many are:

4.	$\frac{5}{6}$ of 12?	$\frac{5}{6}$ of 18?	$\frac{3}{8}$ of 32?	$\frac{3}{4}$ of 8?
5.	$\frac{3}{4}$ of 36?	$\frac{5}{8}$ of 40?	$\frac{4}{5}$ of 10?	$\frac{4}{5}$ of 25?
6.	$\frac{7}{8}$ of 56?	$\frac{2}{5}$ of 10?	$\frac{3}{5}$ of 25?	$\frac{5}{9}$ of 27?
7.	$\frac{5}{8}$ of 24?	$\frac{2}{3}$ of 27?	$\frac{3}{5}$ of 50?	$\frac{5}{9}$ of 45?
8.	$\frac{2}{3}$ of 30?	$\frac{5}{9}$ of 63?	$\frac{3}{7}$ of 56?	$\frac{6}{7}$ of 56?

9. $\frac{2}{3} \times 6 = \frac{2}{3}$ of $6 = 4$. Then $\frac{2}{3} \times 12 =$ _?_ = _?_. $\frac{3}{4} \times 12 =$ _?_ = _?_. $\frac{4}{5} \times 10 =$ _?_ = _?_. $\frac{3}{8} \times 32 =$ _?_ = _?_.

Find:

10.	$\frac{1}{6} \times 12$	$\frac{3}{6} \times 12$	$\frac{4}{6} \times 12$	$\frac{5}{6} \times 12$
11.	$\frac{1}{6} \times 18$	$\frac{2}{6} \times 18$	$\frac{4}{6} \times 18$	$\frac{5}{6} \times 18$
12.	$\frac{1}{5} \times 10$	$\frac{2}{5} \times 10$	$\frac{3}{5} \times 10$	$\frac{4}{5} \times 10$
13.	$\frac{1}{8} \times 56$	$\frac{3}{8} \times 56$	$\frac{5}{8} \times 56$	$\frac{7}{8} \times 56$
14.	$\frac{1}{3} \times 27$	$\frac{2}{3} \times 27$	$\frac{1}{3} \times 300$	$\frac{2}{3} \times 300$

15.	$\frac{1}{4} \times 64$	$\frac{2}{4} \times 64$	$\frac{1}{4} \times 100$	$\frac{3}{4} \times 100$
16.	$\frac{1}{9} \times 63$	$\frac{2}{9} \times 63$	$\frac{4}{9} \times 63$	$\frac{8}{9} \times 63$
17.	$\frac{1}{7} \times 35$	$\frac{2}{7} \times 35$	$\frac{4}{7} \times 35$	$\frac{6}{7} \times 35$
18.	$\frac{1}{10} \times 50$	$\frac{3}{10} \times 50$	$\frac{7}{10} \times 50$	$\frac{9}{10} \times 50$
19.	$\frac{1}{5} \times 65$	$\frac{2}{5} \times 65$	$\frac{3}{5} \times 65$	$\frac{4}{5} \times 65$
20.	$\frac{1}{9} \times 27$	$\frac{2}{9} \times 27$	$\frac{4}{9} \times 27$	$\frac{6}{9} \times 27$

21. Look at your answers for number 20. Is $\frac{4}{9} \times 27$ twice as much as $\frac{2}{9} \times 27$? Should it be? Is $\frac{6}{9} \times 27$ three times as much as $\frac{2}{9} \times 27$? Should it be?

Like Fractions and Unlike Fractions

Study the pairs of like numbers below. Think how these numbers can be written without using the words.

3 tens and 5 tens 1 fifth and 2 fifths 3 eighths and 1 eighth

1. Can you write tens by using place value?
2. Can you write fifths and eighths by using place value?
3. How do you show the value of the fractional parts you are using? What must be alike about two fractions that show the same kind of fractional parts?

Fractions which have the same denominator are called like fractions.
Fractions which have different denominators are called unlike fractions.

4. Tell which of the following pairs are like fractions.

$\frac{4}{5}$ and $\frac{1}{5}$ $\frac{2}{3}$ and $\frac{3}{6}$ $\frac{3}{6}$ and $\frac{4}{6}$ $\frac{3}{4}$ and $\frac{3}{5}$

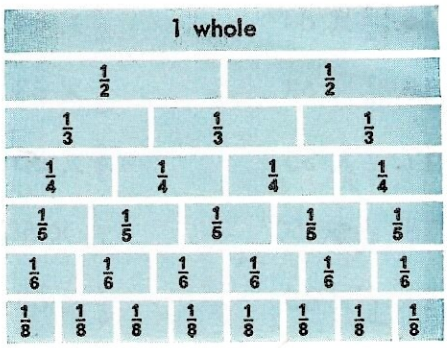
5. Tell which of the following pairs are unlike fractions.

$\frac{1}{2}$ and $\frac{1}{4}$ $\frac{2}{5}$ and $\frac{3}{5}$ $\frac{4}{6}$ and $\frac{2}{3}$ $\frac{1}{8}$ and $\frac{4}{8}$

Comparing Fractions

In her notebook, Betty made a chart to show how certain fractions compare in size. Why did Betty make all the strips in her chart the same size?

When the wholes are the same size, we may compare the fractions.



1. Look at Betty's chart and tell which fraction is larger.

$\frac{2}{4}$ or $\frac{3}{4}$ $\frac{1}{5}$ or $\frac{3}{5}$ $\frac{3}{6}$ or $\frac{5}{6}$

Did you notice that when two fractions have the same denominator, the fraction with the larger numerator has the larger value? Why is this true?

2. Look at the chart again. Tell which is larger.

$\frac{1}{2}$ or $\frac{1}{4}$ $\frac{1}{4}$ or $\frac{1}{3}$ $\frac{1}{3}$ or $\frac{1}{6}$ $\frac{1}{6}$ or $\frac{1}{4}$

$\frac{1}{6}$ or $\frac{1}{2}$ $\frac{1}{4}$ or $\frac{1}{5}$ $\frac{1}{2}$ or $\frac{1}{3}$ $\frac{1}{8}$ or $\frac{1}{6}$

Did you notice that when two fractions have the same numerator, the fraction with the larger denominator has the smaller value? Why is this true?

3. Tell which is larger in each of the following pairs of fractions. Use Betty's chart to help you.

$\frac{1}{2}$ or $\frac{2}{3}$ $\frac{1}{3}$ or $\frac{2}{4}$ $\frac{2}{5}$ or $\frac{3}{6}$ $\frac{4}{5}$ or $\frac{3}{4}$

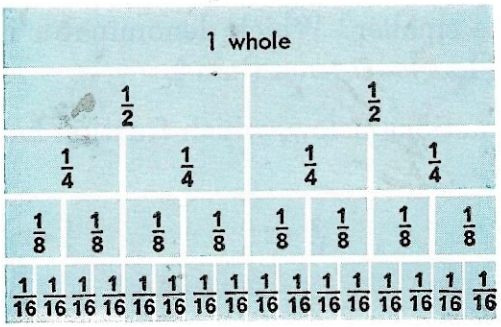
$\frac{2}{3}$ or $\frac{3}{4}$ $\frac{3}{6}$ or $\frac{3}{8}$ $\frac{5}{8}$ or $\frac{5}{6}$ $\frac{2}{5}$ or $\frac{2}{4}$

4. What do we call fractions which have the same denominators? Different denominators? What kind of fractions are those in each pair in exercise 3?

Unlike Fractions of Equal Value

Ned had a chart in his notebook for comparing unlike fractions. He noticed that sometimes unlike fractions of the same whole may be equal in value.

What do you notice on Ned's chart about $\frac{1}{2}$ and $\frac{2}{4}$?



If you need to, use Ned's chart to help you answer the questions which follow.

1. One whole = _?_ halves; _?_ fourths; _?_ eighths; _?_ sixteenths. $1 = \frac{?}{2}; \frac{?}{4}; \frac{?}{8}; \frac{?}{16}$. Are these unlike fractions? Why? Are they equal in value?

2. One half = _?_ fourths; _?_ eighths; _?_ sixteenths. $\frac{1}{2} = \frac{?}{4}; \frac{?}{8}; \frac{?}{16}$. Are these unlike fractions? Why? Are they equal in value?

3. One fourth = _?_ eighths; _?_ sixteenths. $\frac{1}{4} = \frac{?}{8}; \frac{?}{16}$.

4. One eighth = _?_ sixteenths. $\frac{1}{8} = \frac{?}{16}$.

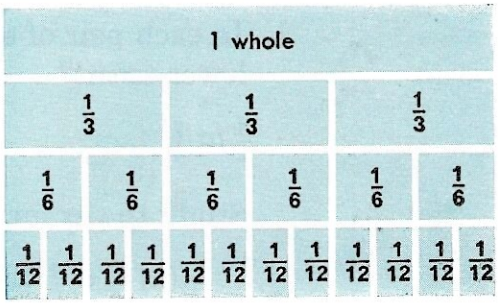
5. Copy and complete. Use Ned's chart to help if necessary.

$\frac{2}{4} = \frac{?}{8}$; $\frac{3}{8} = \frac{?}{16}$; $\frac{3}{4} = \frac{?}{8}$; $\frac{5}{8} = \frac{?}{16}$;

$\frac{8}{16} = \frac{?}{8}$; $\frac{2}{8} = \frac{?}{16}$; $\frac{4}{8} = \frac{?}{16}$; $\frac{6}{8} = \frac{?}{16}$;

$\frac{7}{8} = \frac{?}{16}$; $\frac{10}{16} = \frac{?}{8}$

Ned might have had this chart.



6. One whole = _?_ thirds; _?_ sixths; _?_ twelfths. $1 = \frac{?}{3}; \frac{?}{6}; \frac{?}{12}$. Are these unlike fractions? Why? Are they equal in value?

7. One third = _?_ sixths; _?_ twelfths. $\frac{1}{3} = \frac{?}{6}; \frac{?}{12}$. Are these unlike fractions? Why? Are they equal in value?

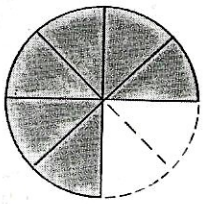
8. One sixth = _?_ twelfths. $\frac{1}{6} = \frac{?}{12}$.

9. Complete. Use the drawing to help if necessary.

$\frac{2}{3} = \frac{?}{12}$; $\frac{12}{12} = \frac{?}{6}$; $\frac{8}{12} = \frac{?}{6}$; $\frac{6}{6} = \frac{?}{3}$;

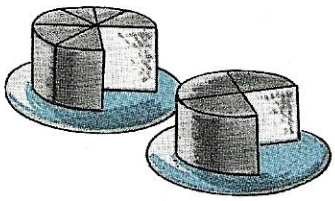
$\frac{3}{6} = \frac{?}{12}$; $\frac{4}{12} = \frac{?}{6}$; $\frac{5}{6} = \frac{?}{12}$; $\frac{10}{12} = \frac{?}{6}$

Changing a Fraction to Lower Terms



The cake at the left has been cut into 8 equal pieces and 2 pieces have been eaten. How much of the cake is left?

Bill said that $\frac{6}{8}$ of the cake was left. Joe said that $\frac{3}{4}$ of the cake was left.



Were both boys correct? Look at the drawings at the left. The two cakes are the same size. Are $\frac{3}{4}$ and $\frac{6}{8}$ unlike fractions of equal value?

Which numerator is the smaller? Which denominator is the smaller? We say that $\frac{3}{4}$ has *lower terms* than $\frac{6}{8}$.

1. In each pair of equal fractions below, which fraction has lower terms?

$\frac{1}{2}$ or $\frac{2}{4}$ $\qquad$ $\frac{4}{6}$ or $\frac{2}{3}$ $\qquad$ $\frac{9}{12}$ or $\frac{3}{4}$ $\qquad$ $\frac{6}{8}$ or $\frac{3}{4}$

Study the examples below.

(a) $\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$ $\qquad$ (b) $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$

Does $\frac{3}{6} = \frac{1}{2}$? Does $\frac{6}{8} = \frac{3}{4}$? Do you change the value of a fraction when you divide both terms by the same number?

In example (a) what number was used to divide both terms? Why was this number chosen? In example (b) what number was used to divide both terms? Why?

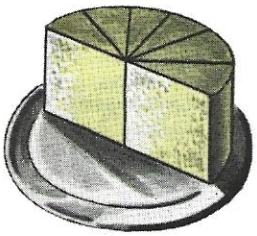
> To change a fraction to lower terms, divide both numerator and denominator by the same number.

2. Change these fractions to lower terms. Remember to divide both terms (numerator and denominator) by the same number.

$\frac{7}{14}$ $\quad$ $\frac{10}{15}$ $\quad$ $\frac{3}{9}$ $\quad$ $\frac{2}{10}$ $\quad$ $\frac{3}{15}$ $\quad$ $\frac{2}{6}$

$\frac{5}{10}$ $\quad$ $\frac{2}{12}$ $\quad$ $\frac{2}{4}$ $\quad$ $\frac{2}{20}$ $\quad$ $\frac{3}{21}$ $\quad$ $\frac{6}{21}$

Changing Fractions to Lowest Terms



The cake at the right has been cut into 12 equal parts and 6 pieces have been served. What fraction of the cake is left?

Bill said that $\frac{6}{12}$ of the cake was left.
Joe said that $\frac{3}{6}$ of the cake was left.
Jim said that $\frac{1}{2}$ of the cake was left.
Were all the boys correct? Are $\frac{6}{12}$, $\frac{3}{6}$, and $\frac{1}{2}$ equal fractions? Which of the three fractions has the lowest terms?

Study the two ways of changing $\frac{6}{12}$ and $\frac{4}{16}$ to lowest terms:

Long Way		*Short Way*
$\frac{6}{12} = \frac{6 \div 2}{12 \div 2} = \frac{3}{6}$	$\frac{3 \div 3}{6 \div 3} = \frac{1}{2}$	$\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$
$\frac{4}{16} = \frac{4 \div 2}{16 \div 2} = \frac{2}{8}$	$\frac{2 \div 2}{8 \div 2} = \frac{1}{4}$	$\frac{4}{16} = \frac{4 \div 4}{16 \div 4} = \frac{1}{4}$

Why is $\frac{3}{6}$ not in lowest terms? Why is $\frac{2}{8}$ not in lowest terms?

> A fraction is in lowest terms when no number except 1 will divide both terms exactly.

What is the largest number that will divide both 6 and 12 evenly? What is the largest number that will divide both 4 and 16 evenly?

When changing a fraction to lowest terms, it will save work if you divide both terms by the largest number that will divide both evenly.

Change the following fractions to lowest terms.

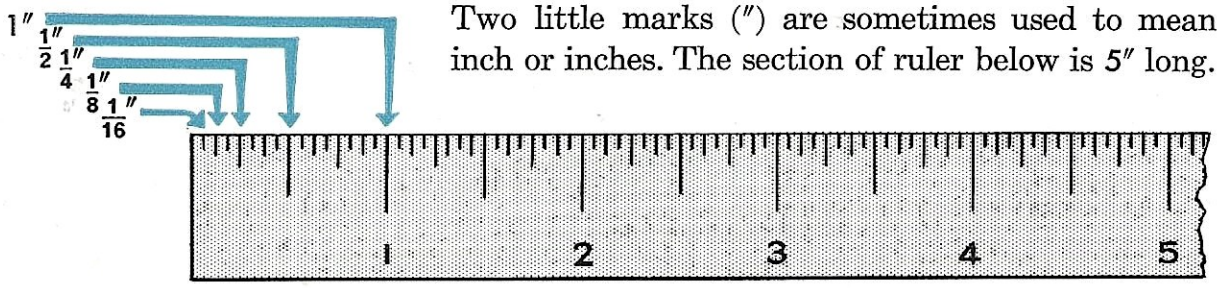
1. $\frac{12}{16}$ $\frac{4}{12}$ $\frac{8}{12}$ $\frac{16}{24}$ $\frac{4}{16}$ $\frac{8}{16}$ $\frac{4}{8}$ $\frac{6}{12}$

2. $\frac{8}{24}$ $\frac{12}{24}$ $\frac{6}{18}$ $\frac{12}{18}$ $\frac{6}{10}$ $\frac{10}{20}$ $\frac{12}{20}$ $\frac{10}{30}$

3. Compare your answers to several of the exercises above with the original fraction in each case. Does the fraction in lowest terms stand for fewer parts of larger size?

Fractions on a Ruler

The fractions on a ruler can help you learn more about fractions.



Two little marks (″) are sometimes used to mean inch or inches. The section of ruler above is 5″ long.

1. Using the ruler to help, count by $\frac{1}{2}''$ to 5″. Count $\frac{1}{2}''$, $\frac{2}{2}''$, $\frac{3}{2}''$, ... Now count $\frac{1}{2}''$, 1″, $1\frac{1}{2}''$, ... How many inches will $\frac{4}{2}''$ make? $\frac{6}{2}''$? $\frac{10}{2}''$?

2. Count backward by $\frac{1}{2}''$ from 3″. $3'' = \frac{?}{2}''$. Begin $\frac{6}{2}''$, $\frac{5}{2}''$, $\frac{4}{2}''$, ... Now count 3″, $2\frac{1}{2}''$, 2″, ... $2'' = \frac{?}{2}''$, $1\frac{1}{2}'' = \frac{?}{2}''$.

3. Use the ruler to show that both of the following ways of counting by $\frac{1}{8}$ to 1 are correct.

$\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{8}{8}$. $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, 1.

This also shows that $\frac{2}{8} = \frac{?}{4}$; $\frac{4}{8} = \frac{?}{2}$; $\frac{6}{8} = \frac{?}{4}$; $\frac{8}{8} = \frac{?}{\ }$.

You may think of adding as a way to put numbers together, or you may think of adding as a short way of counting forward. Adding on a ruler is like counting forward.

4. Add $\frac{1}{8}$ inch and $\frac{4}{8}$ inch. Put your finger on $\frac{1}{8}''$ and count forward $\frac{4}{8}''$. $\frac{1}{8}'' + \frac{4}{8}'' = \frac{?}{8}''$.

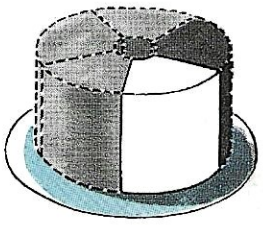
5. Add $\frac{3}{4}$ inch and $\frac{2}{4}$ inch. Put your finger on $\frac{3}{4}''$ and count forward $\frac{2}{4}''$. $\frac{3}{4}'' + \frac{2}{4}'' = \frac{?}{4}'' = 1\frac{?}{4}''$.

6. Find $\frac{1}{16}''$ on your ruler. Count by $\frac{1}{16}''$ to 1″. Add $\frac{3}{16}''$ and $\frac{7}{16}''$. Put your finger on $\frac{3}{16}''$ and count forward $\frac{7}{16}''$. $\frac{3}{16}'' + \frac{7}{16}'' = \frac{?}{16}'' = \frac{?}{8}''$.

7. Add $\frac{1}{8}'' + \frac{3}{8}'' + \frac{1}{8}''$.

8. Add $\frac{1}{8}'' + \frac{3}{8}'' + \frac{2}{8}''$. $\frac{6}{8}'' = \frac{?}{4}''$.

Adding Like Fractions



A small angel food cake was cut into 5 parts. First $\frac{1}{5}$ was served. Then $\frac{3}{5}$ was served. What fraction of the cake was served?

To find how much in all, add $\frac{1}{5}$ and $\frac{3}{5}$. $\frac{1}{5}$ and $\frac{3}{5}$ are like fractions. Like fractions can be added just as any like numbers are added.

1 ten	1 one	1 tenth	1 fifth
3 tens	3 ones	3 tenths	3 fifths
4 tens	4 ones	4 tenths	4 fifths

1 fifth + 3 fifths = _?_ fifths. Four fifths (or $\frac{4}{5}$) of the cake was served.

1 fifth
3 fifths
4 fifths

$\frac{1}{5}$
$\frac{3}{5}$
$\frac{4}{5}$

To add like fractions, add the numerators and write the sum over the denominator.

Complete the following.

1. 1 fourth + 2 fourths = _?_ fourths. $\frac{1}{4} + \frac{2}{4} = \frac{?}{4}$

1 fourth	$\frac{1}{4}$
2 fourths	$\frac{2}{4}$
_?_ fourths	$\frac{?}{4}$

2. 1 sixth + 2 sixths + 2 sixths = _?_ sixths. $\frac{1}{6} + \frac{2}{6} + \frac{2}{6} = \frac{?}{6}$

Find the sums. Be sure your answers are in lowest terms.

3.

$\frac{1}{5}$	$\frac{4}{6}$	$\frac{2}{8}$	$\frac{2}{9}$	$\frac{1}{7}$	$\frac{1}{3}$
$\frac{2}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{3}{9}$	$\frac{3}{7}$	$\frac{1}{3}$

4.

$\frac{1}{8}$	$\frac{2}{6}$	$\frac{1}{9}$	$\frac{1}{5}$	$\frac{1}{12}$	$\frac{2}{10}$
$\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{9}$	$\frac{1}{5}$	$\frac{2}{12}$	$\frac{3}{10}$
$\frac{1}{8}$	$\frac{2}{6}$	$\frac{3}{9}$	$\frac{2}{5}$	$\frac{4}{12}$	$\frac{4}{10}$

Practice to Remember

MAINTENANCE TEST ON MEANINGS AND SKILLS (CHAPTERS 1–2)

1. Write the following words in a column: *subtrahend*, *minuend*, *sum*, *dividend*, *multiplier*, *quotient*, *divisor*, and *addend*. Next to each word, put a number from the examples below that will show that you know what the word means.

76 +36 112	376 −124 252	87 93 261 7 83 8,091	38 r4 42)1,600

Copy. Add down. Check by adding up.

2.

472	7,249	85,762	$.07	$ 5.49	$57.76
114	8,695	43	.04	6.23	.42
403	3,724	5,209	.06	2.37	2.76
		6	.09		24.56

Copy in columns and add.

3. 8 + 9 + 6 + 4 + 3 $45.87 + $.04 + $5.76 + $.41

Subtract. Check by adding the subtrahend and the difference.

4.

72	832	240	$5.21	$43.70	$80,000.00
41	722	168	2.50	.54	8,421.76

Multiply. Check by going over your work.

5.

42	87	682	231	708	$56.72
3	93	400	765	870	45

Divide and check.

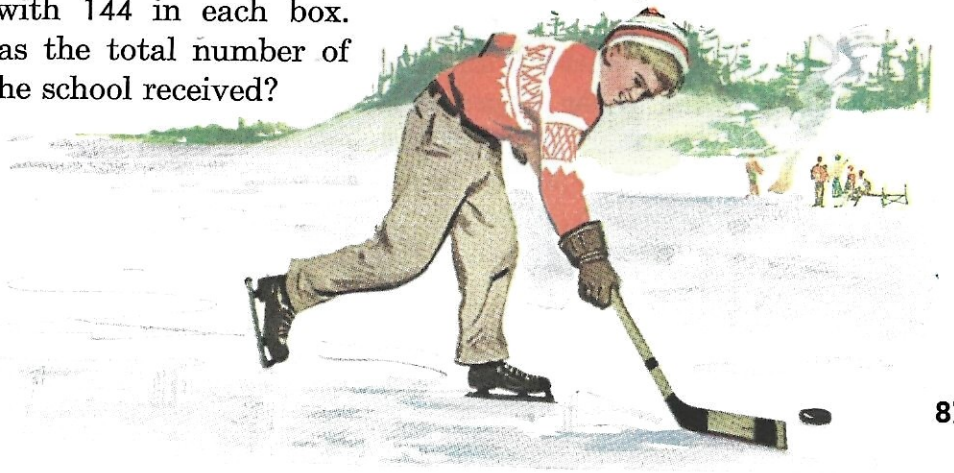
6. 3)936 4)800 5)255 6)427 16)704

7. 19)1064 24)1271 56)3520 86)7749 63)$68.67

Practice to Remember

MAINTENANCE TEST ON PROBLEM SOLVING (CHAPTERS 1–2)

1. Mrs. Green spent $5.78 for groceries, $3.64 for meat, and $2.08 for fruit. How much did she spend in all?
2. If Joe weighs 90 pounds and Tom weighs 78 pounds, how much heavier is Joe than Tom?
3. Ten years ago the population of the town of Chatham was 12,763. Today the population is 16,354. How many more people live in Chatham today than lived there ten years ago?
4. Sue's coat cost $37.00 and Jean's cost $29.98. What is the difference in the price of the coats?
5. At $9.25 a ton, what is the cost of 8 tons of coal?
6. A freight train went 221 miles in 13 hours. What was its average speed?
7. A school received 36 boxes of pencils with 144 in each box. What was the total number of pencils the school received?
8. Mr. Brown sold 189 bushels of apples at $2.37 for each bushel. How much did he get for all the apples?
9. If Carol saves $.15 every day in the month of December, how much money will she have at the end of the month?
10. If 12 pencils cost $.25, how many pencils can you get for $2.00?
11. On Monday Susan practiced her music 30 minutes, on Tuesday 45 minutes, on Wednesday 45 minutes, and on Thursday 40 minutes. What was the average number of minutes Susan practiced her music each day?
12. Bill earned $12.00 shoveling snow. He spent $6.00 for ice skates, $2.98 for gloves, and $3.01 for a hockey stick. How much money did he have left?



Kinds of Numbers

John has a page about different kinds of numbers in his notebook. This is what he has on the page:

Whole numbers are the numbers we used when we first learned to count.

3 5

A *proper fraction* has a numerator that is smaller than the denominator. Its value is less than 1.

$\frac{3}{4}$ $\frac{4}{5}$ $\frac{1}{2}$

An *improper fraction* has a numerator that is the same as or larger than the denominator. Its value is 1 or more than 1.

$\frac{7}{4}$ $\frac{5}{5}$ $\frac{8}{3}$

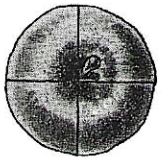
A *mixed number* contains a whole number and a fraction.

$1\frac{1}{2}$ $2\frac{7}{8}$ $1\frac{5}{6}$

1. Make four columns. Label them *whole numbers, proper fractions, improper fractions,* and *mixed numbers.* Then put each of the following under its proper heading.

4	$\frac{2}{5}$	$\frac{5}{8}$	$\frac{19}{5}$	$\frac{14}{3}$	$\frac{3}{8}$
$\frac{18}{9}$	$8\frac{3}{4}$	$\frac{1}{5}$	$5\frac{1}{8}$	$\frac{10}{6}$	11
$\frac{5}{4}$	$\frac{1}{2}$	$16\frac{2}{3}$	19	$\frac{5}{9}$	$6\frac{3}{5}$

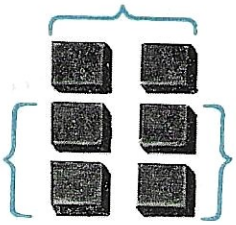
Dividing a Number by a Larger Number



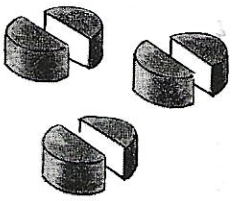
John had an apple that he wished to share equally with his three friends, keeping a share for himself. He divided the apple into 4 equal pieces. Each boy ate $\frac{1}{4}$ of the apple.

$1 \div 4 = \frac{1}{4}$. $\frac{1}{4}$ is another way to write $1 \div 4$ or $4\overline{)1}$.

Any fraction may stand for a division.



Jim wants to share 2 chocolate bars with John and Carl. How many equal shares will he need? He cut each bar into thirds and gave himself and each of the other boys 2 of the pieces. Each boy had $\frac{1}{3}$ of two chocolate bars, or an amount equal to $\frac{2}{3}$ of one chocolate bar. $2 \div 3 = \frac{2}{3}$



1. Ann had 3 cakes to share with 5 friends. How much would each of the 6 girls receive? $3 \div 6 = \frac{?}{6} = \frac{?}{2}$
2. Why was everyone's share less than 1 in each of the examples above?
3. When you divide a number by a larger number, what kind of answer do you get?

When a number is divided by a larger number, the answer is a proper fraction.

4. Draw diagrams if you need help to solve the following. How much will each get if:
   - 3 chocolate bars are shared equally by 4 children?
   - 4 oranges are shared equally by 8 girls?
   - 2 bricks of ice cream are shared equally by 5 boys?

Work these examples. Be sure that each of your answers is in lowest terms.

5. $1 \div 4$ $\quad 4 \div 5$ $\quad 1 \div 6$ $\quad 3 \div 5$ $\quad 8 \div 10$ $\quad 4 \div 20$
6. $1 \div 5$ $\quad 5 \div 6$ $\quad 3 \div 4$ $\quad 2 \div 3$ $\quad 7 \div 14$ $\quad 6 \div 12$

Remainders in Division

1. Three boys want to share 4 candy bars equally. How many candy bars will each boy receive?

To find the answer, divide.

$$\begin{array}{r} 1\frac{1}{3} \\ 3\overline{)4} \\ \underline{3} \\ 1 \end{array} \qquad \begin{array}{r} 1 \text{ r}1 \\ 3\overline{)4} \\ \underline{3} \\ 1 \end{array}$$

Each boy may have 1 candy bar and there will be 1 candy bar left. What will they do with this extra bar? How much is $1 \div 3$?

Which of the two examples at the left shows the more sensible way to answer the question this problem asks?

Each boy will have $1\frac{1}{3}$ candy bars.

Do you see that the remainder is divided by the divisor to give the fraction in the quotient? If the division is correct, will this fraction always be a proper fraction?

2. Four girls wanted to share 10 brownies equally. How many brownies will each girl receive?

$$\begin{array}{r} 2\frac{2}{4} = 2\frac{1}{2} \\ 4\overline{)10} \\ \underline{8} \\ 2 \end{array}$$

To find the answer, divide. If you divide the remainder, the answer in lowest terms is $2\frac{1}{2}$. Is it sensible to say that each girl will receive $2\frac{1}{2}$ brownies?

The exercises on the rest of this page will give you practice in dividing remainders.

3. Study the examples below. Copy them without the work. Divide. Compare your work with the work in the book.

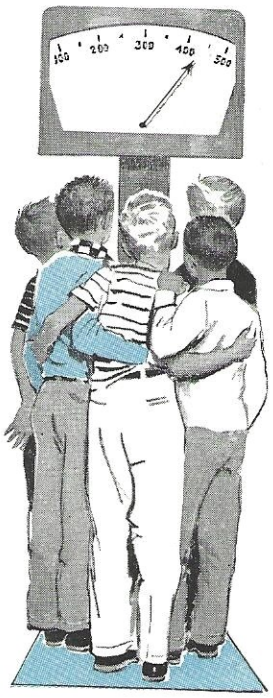
$$\begin{array}{r} 4\frac{5}{7} \\ 7\overline{)33} \\ \underline{28} \\ 5 \end{array} \qquad \begin{array}{r} 7\frac{4}{8} = 7\frac{1}{2} \\ 8\overline{)60} \\ \underline{56} \\ 4 \end{array} \qquad \begin{array}{r} 2\frac{8}{12} = 2\frac{2}{3} \\ 12\overline{)32} \\ \underline{24} \\ 8 \end{array} \qquad \begin{array}{r} 1\frac{13}{24} \\ 24\overline{)37} \\ \underline{24} \\ 13 \end{array} \qquad \begin{array}{r} 6\frac{7}{42} = 6\frac{1}{6} \\ 42\overline{)259} \\ \underline{252} \\ 7 \end{array}$$

Divide. Divide each remainder and express the fraction in lowest terms.

4. $5\overline{)48}$ $6\overline{)40}$ $7\overline{)38}$ $6\overline{)57}$ $9\overline{)75}$ $8\overline{)62}$

5. $11\overline{)47}$ $14\overline{)100}$ $48\overline{)388}$ $24\overline{)54}$ $18\overline{)111}$ $36\overline{)297}$

When you solve a problem in division, you have to think about the sensible way to express the remainder. Solve the following problems. Decide how to deal with the remainder in each of them by thinking what you would actually do in the situation that the problem describes.



6. Five boys together weigh 428 pounds. What is their average weight?

Is it sensible to say that a person's weight might be a certain whole number of pounds and a fraction of a pound?

7. The sixth grade is going to hire buses for a trip to the museum. If there are 138 pupils and each bus will hold 30, how many buses should be hired?

Can you hire a fraction of a bus? Can some of the pupils be left? What will you do with the remainder in answering the question this problem asks?

8. Cans of soup are selling at 2 for 33¢. How much would you have to pay if you bought only 1 can?

Do you do somewhat the same thing with this remainder that you did with the remainder in problem 7?

9. There are 9 players on a baseball team. How many baseball teams can you get from a class of 40 boys?

If you wanted to be fair to everybody, what would you do with the 4 boys who are left after the teams are formed? Should this remainder be expressed as a whole number?

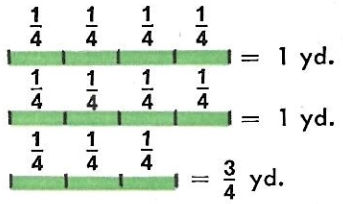
10. How many five-cent candy bars can Jim buy with 79¢? He can buy _?_ candy bars. He will have _?_ ¢ left.

How is your answer to this problem different from your answer to problem 7?

11. The 33 pupils in Miss White's class want to share the cost of an aquarium that costs $5.00. How much should each pay?

Will 15¢ apiece be enough? Will 16¢ apiece make more than $5.00? What do you think the pupils should do?

Changing Improper Fractions to Mixed Numbers or Whole Numbers



Joan is making 11 badges out of ribbon to use at the Pet Show. She needs one quarter yard of ribbon for each badge. She has drawn this picture to show that she needs 11 quarter yards. She will ask the clerk for $2\frac{3}{4}$ yards.

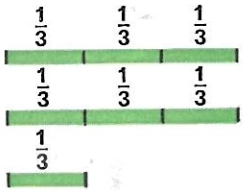
How does the drawing show that $\frac{11}{4}$ yards is equal to $2\frac{3}{4}$ yards?

When you change $\frac{11}{4}$ to $2\frac{3}{4}$, you are changing an improper fraction to a mixed number.

1. Use the drawing to show that $\frac{7}{4} = 1\frac{3}{4}$; $\frac{5}{4} = 1\frac{1}{4}$; $\frac{9}{4} = 2\frac{1}{4}$.

2. Make drawings like the one above to show that:

$$\frac{8}{4} = 2 \qquad \frac{12}{4} = 3$$



When you change $\frac{8}{4}$ to 2, you are changing an improper fraction to a whole number.

3. Explain how the drawing at the left shows that $\frac{7}{3} = 2\frac{1}{3}$.

4. Make drawings to show that $\frac{4}{3} = 1\frac{1}{3}$; $\frac{5}{3} = 1\frac{2}{3}$; $\frac{8}{3} = 2\frac{2}{3}$.

$$\frac{14}{5} = 5\overline{)14}\ \begin{array}{r} 2\frac{4}{5} \\ 10 \\ \hline 4 \end{array}$$

5. Mary says, "When I change $\frac{14}{5}$ to a mixed number, I remember that any fraction may stand for a division. $\frac{14}{5}$ means $14 \div 5$. To have a mixed number, I must divide the remainder."

$$14 \div 5 = 2\tfrac{4}{5}$$ Is $2\frac{4}{5}$ a mixed number?

6. Use Mary's way to change $\frac{18}{6}$ to a whole number; to change $\frac{12}{8}$ to a mixed number. $1\frac{4}{8} = 1\frac{?}{2}$

Fractions which appear in answers are usually changed to lowest terms.

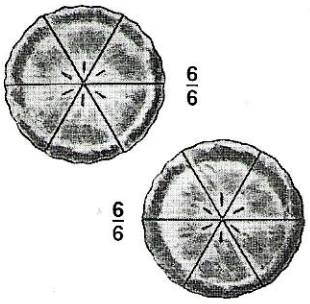
To change an improper fraction to a mixed number or a whole number, divide the numerator by the denominator. Divide any remainder and express the fraction in lowest terms.

7. Change to mixed or whole numbers.

$\frac{7}{4}$ $\frac{16}{8}$ $\frac{11}{6}$ $\frac{14}{4}$ $\frac{12}{5}$ $\frac{36}{12}$ $\frac{7}{7}$ $\frac{22}{6}$

Adding Like Fractions with Sums of One or More than One

CHANGING IMPROPER FRACTIONS TO MIXED NUMBERS OR WHOLE NUMBERS



Mrs. Brown had 2 pies. She cut each pie into 6 equal pieces. How many sixths were in one pie? in the two pies?

6 sixths and 6 sixths = 12 sixths.

Mrs. Brown wanted to serve $\frac{1}{6}$ of a pie to each of 8 boys. That took 8 sixths, which was 1 whole pie and _?_ sixths of the other one. $\frac{8}{6} = 1\frac{2}{6}$

Change $\frac{2}{6}$ to lowest terms. $\frac{2}{6} = \frac{1}{3}$

Mrs. Brown used $1\frac{1}{3}$ pies.

1. The example at the right shows how to add $\frac{5}{6} + \frac{4}{6}$. Think: 5 sixths and 4 sixths are _?_ sixths. $\frac{5}{6} + \frac{4}{6} = \frac{?}{6}$. Change $\frac{9}{6}$ to a mixed number. $9 \div 6 =$ _?_. Change the fraction to lowest terms. $1\frac{3}{6} = 1\frac{?}{2}$

$$\begin{array}{r} \frac{5}{6} \\ \frac{4}{6} \\ \hline \frac{9}{6} \end{array} = 1\frac{3}{6} = 1\frac{1}{2}$$

2. Study the example at the right. $\frac{2}{6} + \frac{3}{6} + \frac{5}{6} = \frac{?}{6}$. Change $\frac{10}{6}$ to a mixed number. $10 \div 6 =$ _?_. Change the fraction to lowest terms. $1\frac{4}{6} = 1\frac{?}{3}$

$$\begin{array}{r} \frac{2}{6} \\ \frac{3}{6} \\ \frac{5}{6} \\ \hline \frac{10}{6} \end{array} = 1\frac{4}{6} = 1\frac{2}{3}$$

Copy and find the sums. Make drawings if you need them. Change improper fractions in the sums to whole or mixed numbers. Change proper fractions to lowest terms.

3.

$\frac{2}{3}$	$\frac{7}{8}$	$\frac{1}{4}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{5}{7}$	$\frac{5}{8}$	$\frac{3}{10}$	$\frac{7}{16}$
$\frac{2}{3}$	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{6}{8}$	$\frac{5}{7}$	$\frac{3}{8}$	$\frac{9}{10}$	$\frac{15}{16}$

4.

$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{7}{9}$	$\frac{8}{15}$	$\frac{3}{10}$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{4}{9}$	$\frac{11}{15}$	$\frac{9}{10}$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{1}{9}$	$\frac{2}{15}$	$\frac{4}{10}$

Adding Whole Numbers, Mixed Numbers, and Fractions

In addition, we may combine only like quantities. We add tens with other tens, ones with other ones, fourths with other fourths, tenths with other tenths, and so forth.

Study the following examples.

3 tens 2 ones and 1 fifth	$32\frac{1}{5}$
$+$4 tens 7 ones and 3 fifths	$+47\frac{3}{5}$
7 tens 9 ones and 4 fifths	$79\frac{4}{5}$

1. Why can we not write 7 tens 9 ones and 4 fifths as 794? Do common fractions have a place value in our number system? In all kinds of number work, we may change the form of any number as long as we do not change its value.

Study the following examples.

5 tens 6 ones and 1 fourth	$56\frac{1}{4}$
$+$2 tens 7 ones and 1 fourth	$+27\frac{1}{4}$
7 tens 13 ones and 2 fourths	$83\frac{2}{4} = 83\frac{1}{2}$
or 8 tens 3 ones and 1 half	
1 ten 9 ones and 5 eighths	$19\frac{5}{8}$
$+$ 3 ones and 3 eighths	$+3\frac{3}{8}$
1 ten 12 ones and 8 eighths	$22\frac{8}{8} = 22 + 1 = 23$
or 2 tens 3 ones	

2. What changes were made in the form of the answers to the examples above? Was the value of any answer changed? How do we decide when to change the form of the answer?

Copy each example below and add. Then change the form of the answer, if necessary, so that any fraction will be a proper fraction in lowest terms.

3.

$8\frac{1}{5}$	$5\frac{1}{4}$	$5\frac{5}{8}$	$4\frac{5}{8}$	$3\frac{3}{4}$	$2\frac{5}{10}$
$\frac{1}{5}$	$1\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{8}$	$1\frac{3}{4}$	$3\frac{7}{10}$

Write only the answers to the following.

4. $4\frac{1}{4}$ $\frac{3}{5}$ 8 $4\frac{5}{8}$ $\frac{2}{8}$ $\frac{2}{5}$ $7\frac{1}{5}$ 3 $1\frac{5}{7}$ 4 $2\frac{3}{5}$ $1\frac{1}{12}$ 6

5. $10 + 1\frac{1}{2}$ $5 + 1\frac{9}{10}$ $3 + 2\frac{1}{5}$ $6 + 1\frac{5}{8}$ $12 + 2\frac{3}{4}$

6. $15 + 1\frac{5}{12}$ $7 + 1\frac{3}{16}$ $8 + 1\frac{4}{15}$ $11 + 1\frac{3}{8}$ $4 + 2\frac{1}{6}$

Copy and add. Be sure all fractions in your answers are proper fractions in lowest terms.

7. $4\frac{1}{4}$ $6\frac{5}{12}$ $1\frac{3}{10}$ $2\frac{1}{5}$ $9\frac{1}{8}$ $2\frac{3}{16}$ $3\frac{1}{6}$
$3\frac{1}{4}$ $\frac{1}{12}$ $11\frac{3}{10}$ $3\frac{1}{5}$ $1\frac{5}{8}$ $\frac{5}{16}$ $\frac{3}{6}$

8. $6\frac{2}{3}$ 4 $3\frac{1}{8}$ $9\frac{1}{6}$ $2\frac{1}{5}$ $3\frac{6}{8}$ 5
4 $2\frac{1}{6}$ $2\frac{5}{8}$ 3 $1\frac{2}{5}$ $\frac{1}{8}$ $9\frac{3}{16}$
1 $3\frac{3}{6}$ 6 $\frac{1}{6}$ $\frac{1}{5}$ 4 2

9. $3\frac{4}{8}$ $1\frac{1}{4}$ $9\frac{1}{8}$ $8\frac{2}{3}$ $4\frac{5}{8}$ $3\frac{4}{9}$ $1\frac{2}{5}$
$2\frac{5}{8}$ $7\frac{3}{4}$ $\frac{7}{8}$ $5\frac{1}{3}$ $2\frac{6}{8}$ $2\frac{5}{9}$ $6\frac{4}{5}$

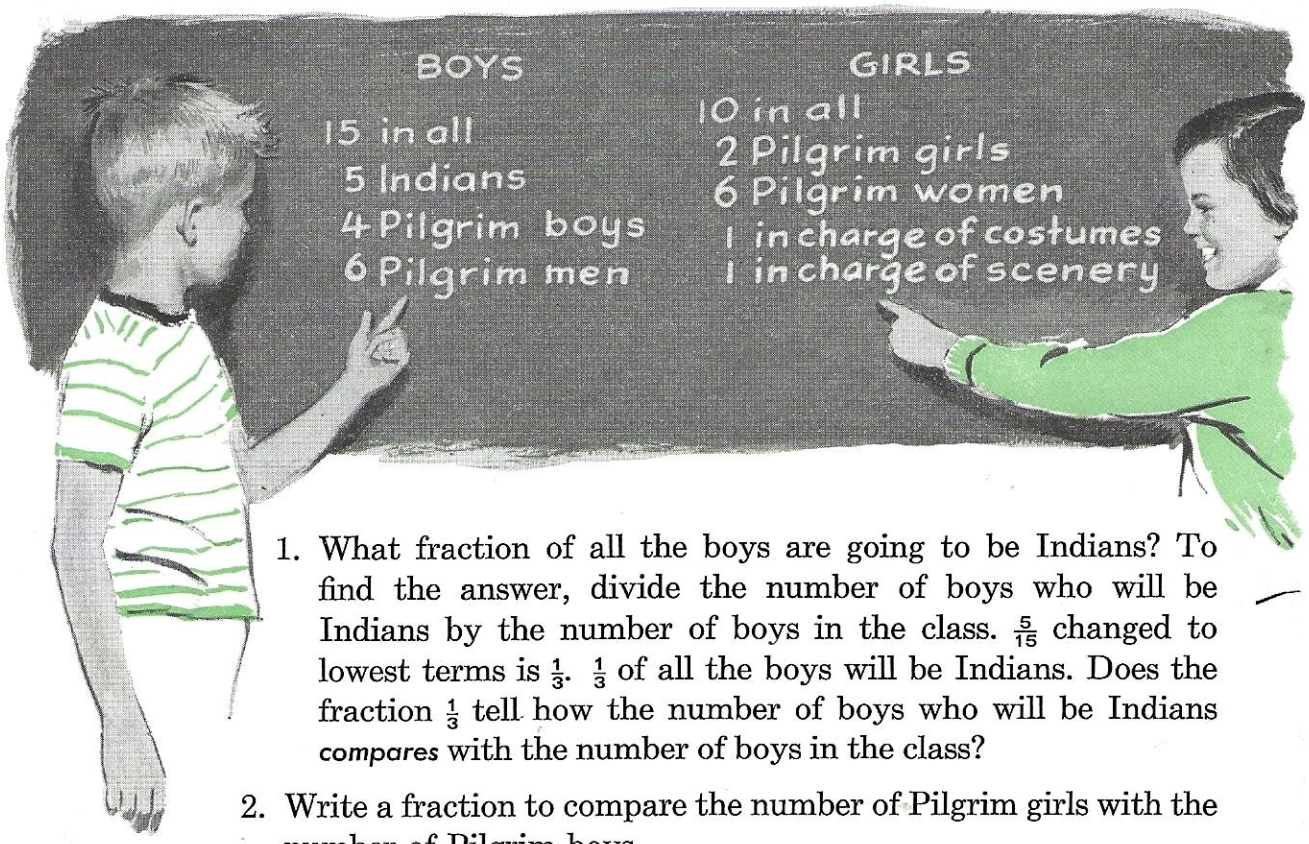
10. $6\frac{5}{12}$ $19\frac{3}{5}$ $3\frac{2}{4}$ $1\frac{9}{10}$ $5\frac{4}{6}$ $10\frac{7}{16}$ $3\frac{1}{2}$
$2\frac{7}{12}$ $2\frac{2}{5}$ $5\frac{3}{4}$ $5\frac{4}{10}$ $3\frac{2}{6}$ $8\frac{12}{16}$ $5\frac{1}{2}$

11. $8\frac{2}{3}$ $3\frac{5}{6}$ $5\frac{4}{5}$ $2\frac{5}{7}$ $1\frac{1}{6}$ $3\frac{1}{4}$ $4\frac{8}{15}$
7 $2\frac{1}{6}$ 1 $6\frac{2}{7}$ $3\frac{5}{6}$ $2\frac{1}{4}$ $6\frac{1}{15}$
$1\frac{1}{3}$ $\frac{1}{6}$ $2\frac{3}{5}$ $\frac{4}{7}$ $2\frac{1}{6}$ $5\frac{3}{4}$ $2\frac{7}{15}$

12. $9\frac{3}{4}$ $5\frac{9}{10}$ $7\frac{5}{12}$ $6\frac{7}{8}$ $1\frac{5}{6}$ $4\frac{13}{20}$ $2\frac{7}{16}$
$8\frac{3}{4}$ $4\frac{3}{10}$ $8\frac{11}{12}$ $6\frac{7}{8}$ $12\frac{5}{6}$ $12\frac{17}{20}$ $3\frac{15}{16}$

A Fraction May Show Comparison

The boys and girls in one of the sixth grades were making plans for a Thanksgiving play. On the blackboard they have written the information you will need to answer the questions below.



1. What fraction of all the boys are going to be Indians? To find the answer, divide the number of boys who will be Indians by the number of boys in the class. $\frac{5}{15}$ changed to lowest terms is $\frac{1}{3}$. $\frac{1}{3}$ of all the boys will be Indians. Does the fraction $\frac{1}{3}$ tell how the number of boys who will be Indians *compares* with the number of boys in the class?

2. Write a fraction to compare the number of Pilgrim girls with the number of Pilgrim boys.

$$\text{Compare: } \frac{\text{Number of Pilgrim girls}}{\text{Number of Pilgrim boys}} = \frac{2}{4} = \frac{?}{?}$$

The answer may be read in two ways: (1) There are half as many Pilgrim girls as Pilgrim boys. (2) There is 1 Pilgrim girl for every 2 Pilgrim boys.

When we compare the smaller of two numbers with the larger by dividing, the answer will be a proper fraction.

3. Write a fraction to compare the number of girls who are to be Pilgrim women with the number of girls in the class. Read your answer in two ways.
4. Write a fraction to compare the number of Pilgrim boys with the number of Pilgrim men. Read it in two ways.
5. Write a fraction to compare the number of Pilgrim boys with the number of Pilgrim girls.

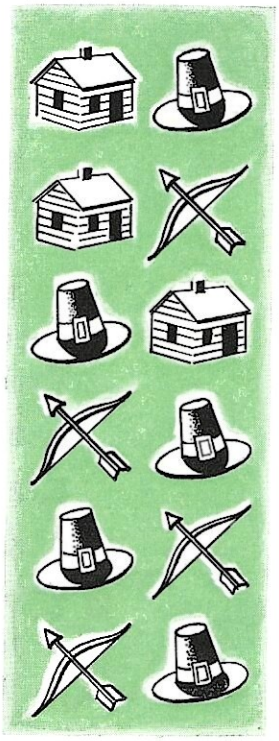
$$\text{Compare: } \frac{\text{Number of Pilgrim boys}}{\text{Number of Pilgrim girls}} = \frac{4}{2} = \frac{2}{1} = 2$$

The answer may be read in two ways: (1) There are twice as many Pilgrim boys as Pilgrim girls. (2) There are 2 Pilgrim boys for every Pilgrim girl.

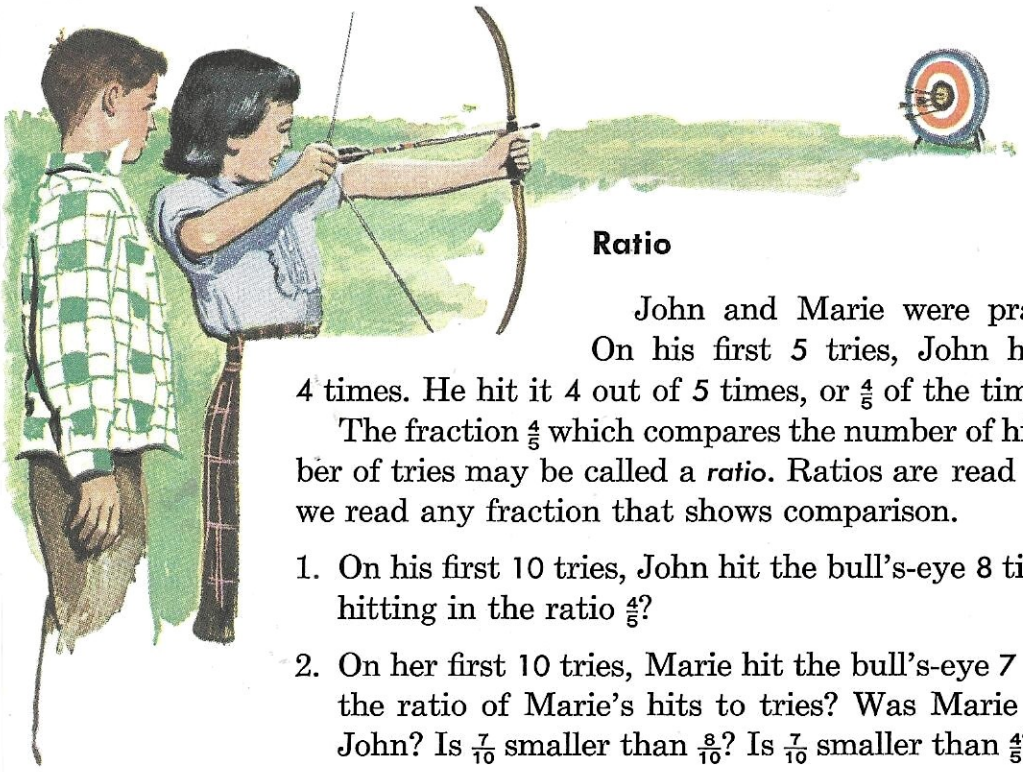
When we compare the larger of two numbers with the smaller by dividing, the answer will be an improper fraction.

6. Write a fraction to compare the number of girls in all with the number of Pilgrim girls. Read your answer in two ways.
7. Write a fraction to compare the number of Pilgrim men with the number of Pilgrim women. How will you read this answer?

Some decorations were to be put on the curtains. Each pupil made a set of decorations like those in the picture at the right.



8. Write a fraction to compare the number of log cabins with the number of Pilgrim hats. Read it in two ways.
9. Write a fraction to compare the number of bows and arrows with the total number of decorations. Read it in two ways.
10. Make up some questions about the decorations or the play and answer your questions.



Ratio

John and Marie were practicing archery. On his first 5 tries, John hit the bull's-eye 4 times. He hit it 4 out of 5 times, or $\frac{4}{5}$ of the time.

The fraction $\frac{4}{5}$ which compares the number of hits with the number of tries may be called a *ratio*. Ratios are read in the same way we read any fraction that shows comparison.

1. On his first 10 tries, John hit the bull's-eye 8 times. Was he still hitting in the ratio $\frac{4}{5}$?

2. On her first 10 tries, Marie hit the bull's-eye 7 times. What was the ratio of Marie's hits to tries? Was Marie doing as well as John? Is $\frac{7}{10}$ smaller than $\frac{8}{10}$? Is $\frac{7}{10}$ smaller than $\frac{4}{5}$?

3. In the supply cabinet, there are 20 arrows and 4 bows. Expressing these numbers as a ratio, we get $\frac{20}{4}$ or $\frac{5}{1}$. We may say that there are 5 arrows for every bow, or that there are _?_ times as many arrows as bows, or that the ratio of arrows to bows is 5 to 1.

A ratio may be expressed by a proper fraction or an improper fraction. In either case, it is usually expressed in lowest terms. Express the following as fractions in lowest terms.

4. 2 out of 10 — 1 out of 3 — 4 out of 8 — 6 out of 8

5. 2 out of 30 — 6 out of 10 — 10 out of 100 — 50 out of 100

6. 6 for every 4 — 5 for every 4 — 10 for every 5

7. Who would you rather have on your team, someone who has had 10 tries and has hit the target 7 times or someone who has had 100 tries and has hit the target 70 times? Why? Do you see why it may not always be best to express a ratio in lowest terms?

Problems Using Fractions

1. Jane needed $1\frac{1}{2}$ cups of nuts for nut bread and $\frac{1}{2}$ cup of nuts for a salad. How many cups of nuts did Jane need for both?

2. Some boys went on a hike. They walked $2\frac{1}{4}$ miles getting to camp. They came back by a different route, walking only $1\frac{3}{4}$ miles on the return trip. How far did they hike in all?

3. Mother bought $1\frac{1}{2}$ pounds of liver in the store and 4 pounds of roast. How much meat did Mother buy in all?

4. Mrs. Jones wanted to serve an apple pie to 8 people. What part of the pie would each person receive?

5. There are 25 pupils in one of the sixth grade rooms. If $\frac{3}{5}$ of the pupils are boys, how many boys are in the room?

6. If $\frac{2}{5}$ of the pupils in the room are girls, how many girls are in the room?

7. Joan was making two kinds of cookies for a party. The butter cookies that she was making needed 1 cup of shortening and the holiday cookies needed $1\frac{1}{3}$ cups of shortening. How much shortening did Joan need for both recipes?

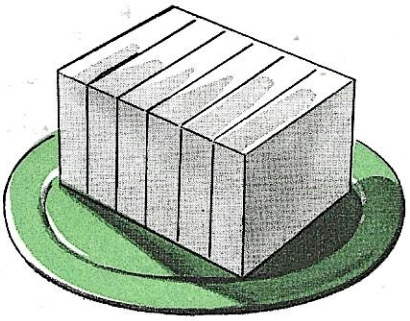
8. Joan's butter cookies needed $\frac{1}{2}$ teaspoon of salt and her holiday cookies needed $1\frac{1}{2}$ teaspoons of salt. How much salt would she need for both recipes?

9. Mrs. Brown made 4 dozen cookies. She put frosting on $\frac{1}{2}$ of the cookies. How many cookies did she put frosting on?

10. Mother made 12 date muffins. If $\frac{2}{3}$ of them were eaten for supper, how many were eaten?

11. Jane bought two remnants of material. In one remnant there were $2\frac{3}{4}$ yards and in the other $2\frac{1}{4}$ yards. How much material did Jane buy?

12. Mary needs 22 inches of elastic to put in the top of her skirt. She found two short pieces in her mother's sewing box. One piece was $10\frac{1}{2}$ inches long and the other piece was $13\frac{1}{4}$ inches long. Her mother said she should allow a total of 1 inch for sewing the pieces together. Will Mary have enough elastic? (Hint: The two pieces together will have to make at least _?_ inches.)



Subtracting Like Fractions

Jerry's mother had a brick of ice cream that was cut into 6 equal parts. She gave Jerry $\frac{1}{6}$ of the brick. What fraction of the brick was left?

To find how much was left, subtract. $\frac{6}{6}$ and $\frac{1}{6}$ are like fractions. Like fractions can be subtracted just as any like things are subtracted.

6 sixths − 1 sixth = ? sixths. ? of the brick was left.

6 sixths
−1 sixth
5 sixths

$\frac{6}{6}$
$-\frac{1}{6}$
$\frac{5}{6}$

To subtract like fractions, subtract the numerators and write the difference over the denominator.

Complete the following.

1. 4 fifths − 1 fifth = ? fifths.

$\frac{4}{5} - \frac{1}{5} = \frac{?}{5}$

4 fifths
−1 fifth
? fifths

$\frac{4}{5}$
$-\frac{1}{5}$
$\frac{?}{5}$

2. 5 sixths − 2 sixths = ? sixths or ? half. $\frac{5}{6} - \frac{2}{6} = \frac{?}{6} = \frac{?}{2}$

5 sixths
−2 sixths
? sixths or ? half

$\frac{5}{6}$
$-\frac{2}{6}$
$\frac{?}{6} = \frac{?}{2}$

3. 7 tenths
−3 tenths
? ? or ? fifths

$\frac{7}{10}$
$-\frac{3}{10}$
$\frac{?}{?} = \frac{?}{5}$

Subtract. Be sure your answers are in lowest terms.

4. $\frac{4}{5} - \frac{1}{5}$ $\frac{8}{9} - \frac{7}{9}$ $\frac{3}{8} - \frac{1}{8}$ $\frac{7}{9} - \frac{4}{9}$

5. $\frac{4}{6} - \frac{1}{6}$ $\frac{5}{8} - \frac{1}{8}$

6. $\frac{7}{10} - \frac{4}{10}$ $\frac{7}{16} - \frac{3}{16}$

7. $\frac{3}{5} - \frac{1}{5}$ $\frac{7}{8} - \frac{5}{8}$

8. $\frac{9}{10} - \frac{7}{10}$ $\frac{9}{16} - \frac{5}{16}$

9. $\frac{7}{12} - \frac{1}{12}$ $\frac{3}{15} - \frac{2}{15}$

10. $\frac{5}{8} - \frac{3}{8}$ $\frac{5}{6} - \frac{1}{6}$

11. $\frac{3}{4} - \frac{1}{4}$ $\frac{11}{8} - \frac{5}{8}$

12. $\frac{8}{8} - \frac{3}{8}$ $\frac{10}{11} - \frac{5}{11}$

13. $\frac{4}{5} - \frac{3}{5}$ $\frac{6}{7} - \frac{5}{7}$

14. $\frac{9}{10} - \frac{1}{10}$ $\frac{14}{9} - \frac{8}{9}$

15. $\frac{4}{7} - \frac{2}{7}$ $\frac{4}{3} - \frac{2}{3}$

16. $\frac{7}{8} - \frac{3}{8}$ $\frac{13}{18} - \frac{7}{18}$

Subtracting Mixed Numbers

Jerry used circles to show how he subtracted $1\frac{1}{4}$ from $2\frac{3}{4}$. He drew 2 circles and $\frac{3}{4}$ of a circle. Then he shaded 1 circle and $\frac{1}{4}$ of a circle. The unshaded part was $1\frac{1}{2}$ circles.

$2\frac{3}{4}$ circles $-$ $1\frac{1}{4}$ circles $=$ $1\frac{1}{2}$ circles

Jerry also subtracted without using his drawing. He wrote the mixed numbers one under the other with fractions under fractions and whole numbers under whole numbers. First he subtracted the fractions, then the whole numbers. He changed the fraction in the answer to lowest terms.

$$\begin{array}{r} 2\frac{3}{4} \\ -1\frac{1}{4} \\ \hline 1\frac{2}{4} = 1\frac{1}{2} \end{array}$$

1. Study the examples at the right. Why is there no fraction in either of the answers?

$$\begin{array}{rr} 5\frac{7}{8} & 1\frac{5}{10} \\ -2\frac{7}{8} & -\frac{5}{10} \\ \hline 3 & 1 \end{array}$$

Copy and subtract. Change proper fractions in the answers to lowest terms.

2.	$4\frac{11}{16}$	$8\frac{2}{3}$	$9\frac{5}{6}$	$6\frac{7}{8}$	$18\frac{3}{4}$	$6\frac{5}{8}$
	$1\frac{5}{16}$	$3\frac{1}{3}$	$5\frac{1}{6}$	$2\frac{3}{8}$	$15\frac{1}{4}$	$1\frac{1}{8}$
3.	$10\frac{5}{6}$	$7\frac{7}{10}$	$6\frac{5}{8}$	$7\frac{1}{4}$	$6\frac{3}{7}$	$9\frac{7}{9}$
	$5\frac{3}{6}$	$2\frac{3}{10}$	$3\frac{1}{8}$	$2\frac{1}{4}$	$\frac{1}{7}$	$\frac{2}{9}$
4.	$6\frac{7}{8}$	$8\frac{2}{3}$	$9\frac{5}{6}$	$12\frac{3}{4}$	$15\frac{4}{5}$	$8\frac{5}{8}$
	$5\frac{3}{8}$	$5\frac{1}{3}$	$4\frac{5}{6}$	$6\frac{1}{4}$	$10\frac{2}{5}$	$1\frac{3}{8}$
5.	$8\frac{4}{5}$	$12\frac{7}{8}$	$9\frac{13}{16}$	$3\frac{5}{6}$	$11\frac{9}{16}$	$7\frac{9}{10}$
	$7\frac{4}{5}$	$\frac{1}{8}$	$\frac{10}{16}$	$\frac{1}{6}$	$\frac{5}{16}$	$\frac{1}{10}$
6.	$5\frac{3}{4}$	$4\frac{1}{8}$	$3\frac{3}{16}$	$2\frac{5}{9}$	$5\frac{2}{7}$	$20\frac{1}{2}$
	$\frac{3}{4}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{5}{9}$	$\frac{2}{7}$	$\frac{1}{2}$

Subtracting a Whole Number from a Mixed Number

Study the two examples at the right; then see if you can do the examples below without help.

$$\begin{array}{r} 5\frac{3}{4} \\ -2 \\ \hline 3\frac{3}{4} \end{array} \qquad \begin{array}{r} 4\frac{3}{8} \\ -4 \\ \hline \frac{3}{8} \end{array}$$

Subtract. Write the answers on folded paper.

1.	$9\frac{11}{16}$	$37\frac{5}{8}$	$20\frac{3}{4}$	$15\frac{3}{5}$	$12\frac{7}{16}$	$14\frac{5}{12}$
	7	17	7	5	9	12
2.	$6\frac{2}{3}$	$25\frac{5}{16}$	$42\frac{3}{10}$	$45\frac{3}{5}$	$12\frac{3}{16}$	$33\frac{5}{12}$
	6	25	15	39	12	3
3.	$16\frac{4}{9}$	$15\frac{5}{12}$	$32\frac{5}{8}$	$24\frac{2}{3}$	$17\frac{11}{12}$	$9\frac{3}{5}$
	16	15	22	24	7	4

Subtracting a Fraction from One

When you subtract a fraction from 1, you have to change the 1 to a fraction so that you will have like fractions to subtract.

1. $1 = \frac{?}{2} = \frac{?}{3} = \frac{?}{4} = \frac{?}{5} = \frac{?}{6} = \frac{?}{7} = \frac{?}{8} = \frac{?}{10} = \frac{?}{16} = \frac{?}{25} = \frac{?}{100} = \frac{?}{1492}$

2. Study the examples at the right. How was the denominator chosen for changing the 1? Copy the examples without the work. Subtract. Check by adding.

$$\begin{array}{r} 1 = \frac{5}{5} \\ -\frac{2}{5} = \frac{2}{5} \\ \hline \frac{3}{5} \end{array} \qquad \begin{array}{r} 1 = \frac{8}{8} \\ -\frac{6}{8} = \frac{6}{8} \\ \hline \frac{2}{8} = \frac{1}{4} \end{array}$$

Copy these examples in vertical form. Subtract and check.

3.	$1 - \frac{1}{2}$	$1 - \frac{3}{6}$	$1 - \frac{3}{4}$	$1 - \frac{2}{7}$	$1 - \frac{3}{5}$	$1 - \frac{2}{3}$
4.	$1 - \frac{4}{5}$	$1 - \frac{5}{8}$	$1 - \frac{3}{10}$	$1 - \frac{6}{16}$	$1 - \frac{9}{10}$	$1 - \frac{15}{16}$
5.	$1 - \frac{1}{5}$	$1 - \frac{3}{7}$	$1 - \frac{7}{8}$	$1 - \frac{1}{4}$	$1 - \frac{1}{6}$	$1 - \frac{7}{10}$
6.	$1 - \frac{3}{40}$	$1 - \frac{7}{32}$	$1 - \frac{2}{11}$	$1 - \frac{21}{50}$	$1 - \frac{37}{100}$	$1 - \frac{29}{1000}$

Subtracting a Fraction or a Mixed Number from a Whole Number

The sixth graders were discussing how to think about problems or examples that contain fractions. Miss White asked them to explain how they would subtract $\frac{3}{4}$ from 3.

Tom said, "I draw a picture like this. I have to cut one circle up into fourths so that I can subtract 3 fourths. I can see that $2\frac{1}{4}$ will be left."

Joan said, "I do what amounts to the same thing, using numbers. I know that I have to have some fourths in the minuend. You have to have fourths if you are going to take away fourths. I can think of the 3 as $2 + 1$, then change the 1 to $\frac{4}{4}$. Now I have $2\frac{4}{4} - \frac{3}{4}$, and that's easy to subtract. The answer is $2\frac{1}{4}$."

$$\begin{array}{r} 3 = 2\frac{4}{4} \\ -\frac{3}{4} = \frac{3}{4} \\ \hline 2\frac{1}{4} \end{array}$$

Arthur said, "I do the same thing as Joan, but I don't think in quite the same way. I see that there is no fraction in the minuend from which to subtract $\frac{3}{4}$, so I have to regroup. I take 1 from the 3, and write the 1 as $\frac{4}{4}$. Now I can subtract."

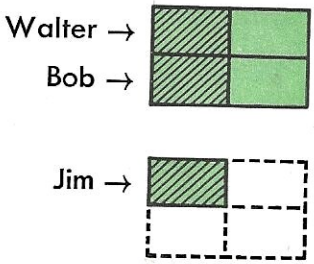
$$\begin{array}{r} 2\frac{4}{4} \\ \cancel{3} \\ -\frac{3}{4} \\ \hline 2\frac{1}{4} \end{array}$$

Use the way you like best to do the following examples. Copy and subtract. Be sure that fractions in your answers are in lowest terms.

1.	3	4	9	6	8	2
	$-\frac{3}{4}$	$-\frac{1}{3}$	$-\frac{2}{5}$	$-\frac{7}{8}$	$-\frac{3}{10}$	$-\frac{7}{12}$
2.	11	14	20	25	16	12
	$-\frac{6}{8}$	$-\frac{4}{10}$	$-\frac{2}{4}$	$-\frac{5}{10}$	$-\frac{2}{8}$	$-\frac{6}{10}$
3.	7	8	4	13	29	10
	$-3\frac{1}{6}$	$-5\frac{2}{3}$	$-2\frac{3}{8}$	$-9\frac{1}{2}$	$-11\frac{1}{3}$	$-6\frac{2}{3}$
4.	5	7	6	17	14	20
	$-1\frac{2}{7}$	$-1\frac{2}{3}$	$-1\frac{5}{6}$	$-5\frac{3}{5}$	$-9\frac{1}{4}$	$-5\frac{5}{8}$

Subtracting a Fraction from a Mixed Number

WITH REGROUPING



Miss White has promised to give Walter, Bob, and Jim each $\frac{1}{4}$ of a large sheet of poster paper. She has $2\frac{1}{4}$ large sheets. If she gives each boy $\frac{1}{4}$ of a sheet, how much paper will be left?

Tom drew a picture. He said his picture shows that Miss White will have $1\frac{1}{2}$ of the large sheets left.

$$\begin{array}{r} 2\frac{1}{4} = 1\frac{5}{4} \\ -\frac{3}{4} = -\frac{3}{4} \\ \hline 1\frac{2}{4} = 1\frac{1}{2} \end{array}$$

Julie said, "We can do this without the drawings. We need more fourths in the minuend, so we think of $2\frac{1}{4}$ first as $1 + 1 + \frac{1}{4}$, then as $1 + \frac{4}{4} + \frac{1}{4}$, then as $1\frac{5}{4}$. Now we can subtract. $1\frac{5}{4} - \frac{3}{4} = 1\frac{2}{4} = 1\frac{1}{2}$."

1. Why did Julie need more fourths in the minuend? Where did she get the $\frac{4}{4}$? the $\frac{5}{4}$? Why did she change $1\frac{2}{4}$ to $1\frac{1}{2}$?

Complete the following.

2. $2\frac{1}{2} = 1 + 1 + \frac{1}{2} = 1 + \frac{2}{2} + \frac{1}{2} = 1\frac{?}{2}$; $\quad 4\frac{1}{3} = 3\frac{?}{3}$;
$2\frac{4}{5} = 1\frac{?}{5}$; $\quad 5\frac{1}{6} = 4\frac{?}{6}$; $\quad 2\frac{3}{8} = 1\frac{?}{8}$; $\quad 5\frac{1}{16} = 4\frac{?}{16}$.

Study these examples. Copy them without the work. Subtract. Compare your answers with those in the book.

3.
$$\begin{array}{r} 1\frac{1}{5} = \frac{6}{5} \\ -\frac{4}{5} = \frac{4}{5} \\ \hline \frac{2}{5} \end{array} \qquad \begin{array}{r} 5\frac{1}{4} = 4\frac{5}{4} \\ -\frac{3}{4} = \frac{3}{4} \\ \hline 4\frac{2}{4} = 4\frac{1}{2} \end{array} \qquad \begin{array}{r} 7\frac{3}{8} = 6\frac{11}{8} \\ -\frac{7}{8} = \frac{7}{8} \\ \hline 6\frac{4}{8} = 6\frac{1}{2} \end{array}$$

Copy and subtract. Check by adding.

4.
$$\begin{array}{r} 1\frac{1}{3} \\ -\frac{2}{3} \\ \hline \end{array} \quad \begin{array}{r} 1\frac{2}{5} \\ -\frac{3}{5} \\ \hline \end{array} \quad \begin{array}{r} 1\frac{3}{7} \\ -\frac{6}{7} \\ \hline \end{array} \quad \begin{array}{r} 1\frac{1}{4} \\ -\frac{2}{4} \\ \hline \end{array} \quad \begin{array}{r} 1\frac{3}{10} \\ -\frac{7}{10} \\ \hline \end{array} \quad \begin{array}{r} 1\frac{5}{12} \\ -\frac{11}{12} \\ \hline \end{array}$$

5.
$$\begin{array}{r} 2\frac{1}{8} \\ -\frac{7}{8} \\ \hline \end{array} \quad \begin{array}{r} 3\frac{3}{5} \\ -\frac{4}{5} \\ \hline \end{array} \quad \begin{array}{r} 6\frac{1}{9} \\ -\frac{7}{9} \\ \hline \end{array} \quad \begin{array}{r} 9\frac{2}{7} \\ -\frac{4}{7} \\ \hline \end{array} \quad \begin{array}{r} 18\frac{1}{6} \\ -\frac{3}{6} \\ \hline \end{array} \quad \begin{array}{r} 12\frac{2}{9} \\ -\frac{8}{9} \\ \hline \end{array}$$

6. $4\frac{1}{4}$ $5\frac{1}{5}$ $8\frac{1}{3}$ $8\frac{1}{10}$ $9\frac{3}{7}$ $7\frac{2}{5}$

$\underline{\frac{3}{4}}$ $\underline{\frac{4}{5}}$ $\underline{\frac{2}{3}}$ $\underline{\frac{3}{10}}$ $\underline{\frac{5}{7}}$ $\underline{\frac{4}{5}}$

Subtracting a Mixed Number from a Mixed Number

WITH REGROUPING

Jane used circles to help her subtract $1\frac{3}{4}$ from $2\frac{1}{4}$. First she drew 2 whole circles and $\frac{1}{4}$ of a circle. Then she shaded $1\frac{3}{4}$ circles to show how much was taken away. How much was left? $\frac{2}{4} = \frac{?}{2}$

Alice did her subtraction without using circles. Her work is shown in the box at the right. Can you explain it?

$2\frac{1}{4} = (1 + \frac{4}{4}) + \frac{1}{4} = 1\frac{5}{4}$

$1\frac{3}{4} =$ $\underline{\qquad\qquad 1\frac{3}{4}}$

$\frac{2}{4} = \frac{1}{2}$

Study the examples below. Copy them without the work. Subtract. Compare your answers with those in the book.

1. $10\frac{2}{5} = 9\frac{7}{5}$ $\quad 5\frac{3}{16} = 4\frac{19}{16}$ $\quad 16\frac{3}{16} = 15\frac{19}{16}$

$\underline{7\frac{4}{5} = 7\frac{4}{5}}$ $\quad \underline{4\frac{9}{16} = 4\frac{9}{16}}$ $\quad \underline{5\frac{7}{16} = 5\frac{7}{16}}$

$2\frac{3}{5}$ $\quad \frac{10}{16} = \frac{5}{8}$ $\quad 10\frac{12}{16} = 10\frac{3}{4}$

Copy and subtract. Check by adding.

2. $14\frac{1}{3}$ $13\frac{3}{8}$ $15\frac{3}{16}$ $14\frac{1}{5}$ $9\frac{2}{7}$ $12\frac{3}{16}$

$\underline{5\frac{2}{3}}$ $\underline{4\frac{5}{8}}$ $\underline{6\frac{9}{16}}$ $\underline{7\frac{3}{5}}$ $\underline{5\frac{5}{7}}$ $\underline{7\frac{7}{16}}$

3. $9\frac{3}{5}$ $7\frac{1}{4}$ $5\frac{5}{8}$ $3\frac{5}{16}$ $2\frac{3}{10}$ $3\frac{1}{9}$

$\underline{8\frac{4}{5}}$ $\underline{6\frac{3}{4}}$ $\underline{4\frac{7}{8}}$ $\underline{2\frac{9}{16}}$ $\underline{1\frac{7}{10}}$ $\underline{2\frac{7}{9}}$

4. Look back at the examples on this page and on the three pages just before it. In these examples, do we change the form of the minuend? Why do we need to do this? Do we change the value of the minuend? Could you say that we regrouped the numbers in the minuend? What did we exchange in this regrouping?

Changing a Fraction to Higher Terms

We know that we may change the form of numbers any way we want to as long as we do not change their value. We may change 7 to $5 + 2$, to $10 - 3$, to $6\frac{2}{2}$, or to hundreds of other groups of numbers. We may change $\frac{2}{10}$ to $\frac{1}{5}$, to $\frac{4}{20}$, to $\frac{20}{100}$, or to any other fraction that has the same value.

1. Write five fractions that have the same value as $\frac{1}{2}$.
2. Look at the fractions $\frac{1}{2}$ and $\frac{5}{10}$. Are these fractions equal in value? Draw a picture to help you if you are not sure. In $\frac{5}{10}$, do you have 5 times as many parts as in $\frac{1}{2}$? How does each part of $\frac{5}{10}$ compare in size with $\frac{1}{2}$? Could you change $\frac{1}{2}$ to $\frac{5}{10}$ by multiplying both terms (numerator and denominator) by 5? $\frac{1 \times 5}{2 \times 5} = \frac{?}{?}$. Could you change $\frac{5}{10}$ to $\frac{1}{2}$ by dividing both terms by 5? $\frac{5 \div 5}{10 \div 5} = \frac{?}{?}$
3. Is the value of a fraction changed if both terms are multiplied by the same number? If both terms are divided by the same number?
4. Would you multiply to make the terms of a fraction larger? Would you divide to make the terms of a fraction smaller?
5. Suppose you want to change $\frac{3}{4}$ to $\frac{?}{24}$. How can you tell what number to multiply by? Think: $4 \times \underline{?} = 24$. In changing 4ths to 24ths, are you changing the fraction to higher terms?

Find the missing numerators.

6. $\frac{1}{5} = \frac{?}{10}$ $\quad \frac{2}{5} = \frac{?}{10}$ $\quad \frac{3}{5} = \frac{?}{10}$
7. $\frac{1}{2} = \frac{?}{10}$ $\quad \frac{1}{2} = \frac{?}{8}$ $\quad \frac{1}{2} = \frac{?}{4}$
8. $\frac{1}{4} = \frac{?}{8}$ $\quad \frac{3}{4} = \frac{?}{8}$ $\quad \frac{3}{4} = \frac{?}{16}$
9. $\frac{1}{6} = \frac{?}{12}$ $\quad \frac{1}{3} = \frac{?}{12}$ $\quad \frac{2}{3} = \frac{?}{12}$
10. $\frac{1}{8} = \frac{?}{16}$ $\quad \frac{3}{8} = \frac{?}{16}$ $\quad \frac{5}{8} = \frac{?}{16}$
11. $\frac{1}{4} = \frac{?}{20}$ $\quad \frac{1}{5} = \frac{?}{20}$ $\quad \frac{1}{2} = \frac{?}{20}$
12. $\frac{5}{5} = \frac{?}{10}$ $\quad \frac{5}{5} = \frac{?}{20}$ $\quad \frac{1}{2} = \frac{?}{6}$
13. $\frac{1}{2} = \frac{?}{16}$ $\quad \frac{3}{4} = \frac{?}{12}$ $\quad \frac{3}{4} = \frac{?}{20}$
14. $\frac{5}{6} = \frac{?}{12}$ $\quad \frac{6}{8} = \frac{?}{16}$ $\quad \frac{7}{8} = \frac{?}{16}$
15. $\frac{1}{4} = \frac{?}{16}$ $\quad \frac{1}{2} = \frac{?}{100}$ $\quad \frac{2}{3} = \frac{?}{30}$

If both terms of a fraction are multiplied by the same number or divided by the same number, the value of the fraction is not changed.

Adding Fractions

UNLIKE FRACTIONS; RELATED

1. In which of the examples at the right can you add without changing the form of the fractions? Why?

2. How can you tell whether fractions are like fractions?

3. Suppose you want to add $\frac{1}{4}$ and $\frac{1}{8}$. What can you do to make them like fractions? Can you change 1 fourth to eighths? How? Can you add 2 eighths and 1 eighth? What is the sum?

4. Suppose you want to add $\frac{2}{3}$ and $\frac{1}{12}$. Can you change 1 twelfth to thirds? Can you change 2 thirds to twelfths? How? Can you add 8 twelfths and 1 twelfth? What is the sum? Is $\frac{9}{12}$ in lowest terms? Change it to lowest terms.

$$\begin{array}{r} \frac{2}{7} \\ +\frac{3}{7} \\ \hline \end{array} \qquad \begin{array}{r} \frac{2}{5} \\ +\frac{3}{10} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{1}{4} = \frac{2}{8} \\ \frac{1}{8} = \frac{1}{8} \\ \hline \frac{3}{8} \end{array}$$

$$\begin{array}{r} \frac{2}{3} = \frac{8}{12} \\ \frac{1}{12} = \frac{1}{12} \\ \hline \frac{9}{12} = \frac{3}{4} \end{array}$$

Copy and add. Be sure your answers are in lowest terms.

5.
$$\begin{array}{r} \frac{1}{2} \\ \frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{4} \\ \frac{1}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{4} \\ \frac{3}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{4} \\ \frac{5}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{2}{4} \\ \frac{1}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{1}{8} \\ \hline \end{array}$$

6.
$$\begin{array}{r} \frac{1}{8} \\ \frac{1}{16} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{16} \\ \frac{1}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{8} \\ \frac{7}{16} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{16} \\ \frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{1}{16} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{3}{16} \\ \hline \end{array}$$

7.
$$\begin{array}{r} \frac{1}{2} \\ \frac{1}{6} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{2} \\ \frac{2}{6} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{3} \\ \frac{1}{6} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{3} \\ \frac{3}{6} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{2} \\ \frac{1}{10} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{2} \\ \frac{3}{10} \\ \hline \end{array}$$

8.
$$\begin{array}{r} \frac{1}{3} \\ \frac{1}{12} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{3} \\ \frac{5}{12} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{3} \\ \frac{7}{12} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{5} \\ \frac{1}{10} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{10} \\ \frac{2}{5} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{10} \\ \frac{3}{5} \\ \hline \end{array}$$

9.
$$\begin{array}{r} \frac{1}{12} \\ \frac{1}{4} \\ \frac{1}{12} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{12} \\ \frac{3}{4} \\ \frac{1}{12} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{4} \\ \frac{5}{12} \\ \frac{1}{12} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{4} \\ \frac{7}{12} \\ \frac{1}{12} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{2} \\ \frac{1}{12} \\ \frac{1}{12} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{12} \\ \frac{1}{2} \\ \frac{1}{12} \\ \hline \end{array}$$

Adding Fractions

UNLIKE FRACTIONS; RELATED; SUMS OF 1 OR MORE THAN 1

$$\begin{array}{r} \frac{3}{8} = \frac{3}{8} \\ \frac{3}{4} = \frac{6}{8} \\ \hline \frac{9}{8} = 1\frac{1}{8} \end{array}$$

1. "You go $\frac{3}{8}$ mile straight down this road from here to the corner. Turn right, and $\frac{3}{4}$ mile from the corner is John's house." How far is it to John's house from here? To find the answer, add $\frac{3}{8}$ and $\frac{3}{4}$. What must you do before you can add them? Look at the example at the left. What kind of number is the sum of $\frac{3}{8}$ and $\frac{6}{8}$? What do you do with improper fractions in your answers? It is $1\frac{1}{8}$ miles to John's house from here.

   Can thirds, halves, and sixths all be written as sixths?
   Can fifths, halves, and tenths all be written as tenths?
   Can halves, fourths, and eighths all be written as eighths?

2. Study the examples below. Copy them without the work. Add. Compare your answers with those in the book.

$$\begin{array}{r} \frac{1}{3} = \frac{2}{6} \\ \frac{1}{2} = \frac{3}{6} \\ \frac{5}{6} = \frac{5}{6} \\ \hline \frac{10}{6} = 1\frac{4}{6} = 1\frac{2}{3} \end{array} \qquad \begin{array}{r} \frac{1}{5} = \frac{2}{10} \\ \frac{1}{2} = \frac{5}{10} \\ \frac{3}{10} = \frac{3}{10} \\ \hline \frac{10}{10} = 1 \end{array} \qquad \begin{array}{r} \frac{1}{2} = \frac{4}{8} \\ \frac{3}{4} = \frac{6}{8} \\ \frac{5}{8} = \frac{5}{8} \\ \hline \frac{15}{8} = 1\frac{7}{8} \end{array}$$

Copy and add. Be sure all fractions in your answers are proper fractions in lowest terms.

3.
$$\begin{array}{r} \frac{5}{8} \\ \frac{3}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{3} \\ \frac{5}{6} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{12} \\ \frac{3}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{3} \\ \frac{7}{9} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{5} \\ \frac{7}{10} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{11}{16} \\ \hline \end{array}$$

4.
$$\begin{array}{r} \frac{1}{2} \\ \frac{1}{4} \\ \frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{5}{8} \\ \frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} \frac{4}{5} \\ \frac{3}{10} \\ \frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{4} \\ \frac{1}{2} \\ \frac{3}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{3}{8} \\ \frac{3}{16} \\ \hline \end{array} \quad \begin{array}{r} \frac{2}{3} \\ \frac{5}{6} \\ \frac{5}{12} \\ \hline \end{array}$$

5.
$$\begin{array}{r} \frac{1}{4} \\ \frac{7}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{2} \\ \frac{5}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{12} \\ \frac{2}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{2}{3} \\ \frac{11}{12} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{8} \\ \frac{5}{16} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{3} \\ \frac{6}{9} \\ \hline \end{array}$$

Adding Mixed Numbers

UNLIKE FRACTIONS; RELATED

1. Joan wants to add $1\frac{3}{8}$ and $2\frac{1}{4}$. She knows that she can add only like fractions. What must she do before she can add? Joan thinks: "I cannot always change eighths to fourths, because fourths are larger than eighths, and I might not always have enough eighths to make a whole number of fourths. I can always change fourths to eighths, because I can always cut up larger parts to make smaller parts. The smaller denominator is the one I must change." The example at the right shows how Joan did her work. Explain it.

$$\begin{aligned} 1\tfrac{3}{8} &= 1\tfrac{3}{8} \\ 2\tfrac{1}{4} &= 2\tfrac{2}{8} \\ \hline & \quad 3\tfrac{5}{8} \end{aligned}$$

2. To add $6\frac{2}{3}$ and $5\frac{5}{6}$, Joan knows that she must change the thirds to sixths. Why? Study her example. How can she change $\frac{9}{6}$ to $1\frac{1}{2}$? What is the final answer?

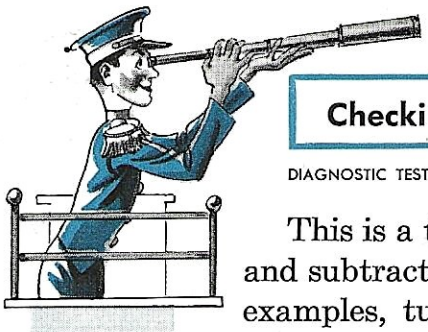
$$\begin{aligned} 6\tfrac{2}{3} &= 6\tfrac{4}{6} \\ 5\tfrac{5}{6} &= 5\tfrac{5}{6} \\ \hline & \quad 11\tfrac{9}{6} = \\ & 11 + 1\tfrac{1}{2} = 12\tfrac{1}{2} \end{aligned}$$

3. Study the examples below. Tell how Joan found her answers. Copy the examples without the work. Add. Compare your answers with those in the book.

$$\begin{aligned} 5\tfrac{2}{5} &= 5\tfrac{4}{10} \\ +6\tfrac{3}{10} &= 6\tfrac{3}{10} \\ \hline & \quad 11\tfrac{7}{10} \end{aligned} \qquad \begin{aligned} 4\tfrac{3}{4} &= 4\tfrac{6}{8} \\ 2\tfrac{5}{8} &= 2\tfrac{5}{8} \\ \hline & \quad 6\tfrac{11}{8} = 6 + 1\tfrac{3}{8} = 7\tfrac{3}{8} \end{aligned} \qquad \begin{aligned} 8\tfrac{3}{4} &= 8\tfrac{3}{4} \\ +2\tfrac{1}{2} &= 2\tfrac{2}{4} \\ \hline & \quad 10\tfrac{5}{4} = 11\tfrac{1}{4} \end{aligned}$$

Copy and add. Be sure all fractions in your answers are proper fractions in lowest terms.

4.	$3\frac{1}{4}$	$6\frac{1}{3}$	$7\frac{2}{3}$	$2\frac{5}{8}$	$7\frac{1}{4}$	$8\frac{1}{2}$
	$5\frac{3}{8}$	$2\frac{5}{6}$	$8\frac{5}{6}$	$3\frac{1}{4}$	$5\frac{1}{2}$	$4\frac{3}{4}$
5.	$7\frac{1}{6}$	$9\frac{1}{8}$	$6\frac{1}{10}$	$7\frac{3}{4}$	$9\frac{7}{8}$	$4\frac{5}{8}$
	$8\frac{2}{3}$	$9\frac{3}{4}$	$8\frac{2}{5}$	$1\frac{1}{2}$	$5\frac{1}{4}$	$7\frac{3}{4}$
6.	$9\frac{5}{6}$	$7\frac{8}{9}$	$8\frac{5}{6}$	$9\frac{1}{5}$	$6\frac{3}{4}$	$8\frac{2}{3}$
	$7\frac{2}{3}$	$5\frac{1}{3}$	$8\frac{7}{12}$	$4\frac{9}{10}$	$7\frac{7}{8}$	$5\frac{5}{6}$



Checking Up on Fractions

DIAGNOSTIC TEST ON PP. 83–109

This is a test to find out if you need more help in adding and subtracting fractions. If you make mistakes in a row of examples, turn to the page shown at the left of the row. Study the explanation and repeat the practice. Then try the row on this page again.

83 1. Change to lowest terms: $\frac{2}{4}$ $\frac{6}{15}$ $\frac{5}{20}$ $\frac{12}{24}$

Copy and add. Change improper fractions in the answers to whole numbers or mixed numbers. Be sure proper fractions are in lowest terms.

85, 93 2.

$\frac{1}{5}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{2}{9}$	$\frac{4}{5}$	$\frac{9}{10}$
$+\frac{3}{5}$	$+\frac{1}{6}$	$+\frac{1}{8}$	$+\frac{4}{9}$	$+\frac{3}{5}$	$+\frac{3}{10}$

94 3.

$2\frac{1}{3}$	8	$4\frac{1}{6}$	$3\frac{2}{3}$	$5\frac{11}{12}$	$6\frac{13}{16}$
$+\frac{1}{3}$	$+\frac{2}{3}$	$+\frac{4}{6}$	$+7\frac{1}{3}$	$+3\frac{5}{12}$	$+9\frac{9}{16}$

107-109 4.

$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{4}$	$6\frac{2}{5}$	$9\frac{1}{6}$	$5\frac{1}{2}$
$+\frac{1}{6}$	$+\frac{2}{10}$	$+\frac{5}{12}$	$+8\frac{1}{10}$	$+2\frac{5}{12}$	$+2\frac{3}{4}$

Subtract. Remember to change proper fractions in the answers to lowest terms.

100, 101 5.

$\frac{2}{3}$	$\frac{9}{10}$	$\frac{8}{9}$	$7\frac{3}{4}$	$6\frac{8}{9}$	$9\frac{5}{12}$
$-\frac{1}{3}$	$-\frac{3}{10}$	$-\frac{3}{9}$	$-2\frac{1}{4}$	$-5\frac{8}{9}$	$-9\frac{1}{12}$

103 6.

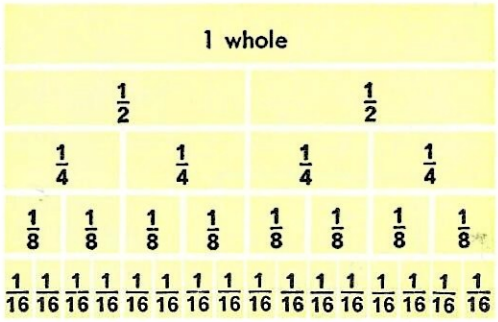
1	1	1	6	7	9
$-\frac{3}{8}$	$-\frac{3}{4}$	$-\frac{7}{10}$	$-\frac{3}{5}$	$-2\frac{7}{8}$	$-4\frac{8}{9}$

104, 105 7.

$1\frac{2}{5}$	$8\frac{2}{7}$	$5\frac{1}{4}$	$4\frac{3}{8}$	$12\frac{2}{7}$	$15\frac{3}{16}$
$-\frac{3}{5}$	$-\frac{4}{7}$	$-\frac{3}{4}$	$-1\frac{5}{8}$	$-6\frac{5}{7}$	$-7\frac{7}{16}$

Subtracting Fractions

UNLIKE FRACTIONS; RELATED



1. You know that unlike fractions must be changed to like fractions before they can be added. Is a similar statement true for subtraction?

To subtract $\frac{1}{8}$ from $\frac{1}{2}$, you would think: 1 half = $\frac{?}{}$ eighths. 4 eighths − 1 eighth = 3 eighths. Show this subtraction using the diagram at the right. Do the same example without using the diagram.

Copy and subtract. Use the diagram for help if you need it.

2. $\frac{3}{4} - \frac{1}{2} = \frac{?}{4}$ $\quad \frac{3}{8} - \frac{1}{4} = \frac{?}{8}$ $\quad \frac{1}{2} - \frac{1}{8} = \frac{?}{8}$ $\quad \frac{1}{2} - \frac{3}{8} = \frac{?}{8}$

3. $\frac{1}{2} - \frac{5}{16} = \frac{?}{16}$ $\quad \frac{1}{8} - \frac{1}{16} = \frac{?}{16}$ $\quad \frac{1}{4} - \frac{1}{16} = \frac{?}{16}$ $\quad \frac{1}{2} - \frac{7}{16} = \frac{?}{16}$

4. $\frac{1}{4} - \frac{1}{8} = \frac{?}{8}$ $\quad \frac{3}{4} - \frac{3}{8} = \frac{?}{8}$ $\quad \frac{5}{8} - \frac{1}{4} = \frac{?}{8}$ $\quad \frac{5}{8} - \frac{1}{2} = \frac{?}{8}$

5. $\frac{1}{2} - \frac{3}{16} = \frac{?}{16}$ $\quad \frac{3}{8} - \frac{3}{16} = \frac{?}{16}$ $\quad \frac{3}{4} - \frac{5}{16} = \frac{?}{16}$ $\quad \frac{9}{16} - \frac{1}{2} = \frac{?}{16}$

6. Study these examples. Then copy them without the work. Subtract. Compare your answers with those in the book.

$\frac{5}{6} = \frac{5}{6}$ $\quad$ $\frac{9}{10} = \frac{9}{10}$ $\quad$ $\frac{2}{3} = \frac{8}{12}$ $\quad$ $\frac{5}{9} = \frac{5}{9}$

$\underline{\frac{1}{3} = \frac{2}{6}}$ $\quad$ $\underline{\frac{1}{5} = \frac{2}{10}}$ $\quad$ $\underline{\frac{5}{12} = \frac{5}{12}}$ $\quad$ $\underline{\frac{1}{3} = \frac{3}{9}}$

$\frac{3}{6} = \frac{1}{2}$ $\quad$ $\frac{7}{10}$ $\quad$ $\frac{3}{12} = \frac{1}{4}$ $\quad$ $\frac{2}{9}$

Copy and subtract. Be sure that your answers are in lowest terms.

7. $\frac{7}{12}$ $\quad \frac{9}{16}$ $\quad \frac{7}{16}$ $\quad \frac{11}{12}$ $\quad \frac{3}{4}$ $\quad \frac{7}{8}$ $\quad \frac{7}{10}$ $\quad \frac{5}{8}$

$\underline{\frac{1}{4}}$ $\quad \underline{\frac{1}{2}}$ $\quad \underline{\frac{1}{4}}$ $\quad \underline{\frac{1}{6}}$ $\quad \underline{\frac{5}{8}}$ $\quad \underline{\frac{1}{4}}$ $\quad \underline{\frac{1}{2}}$ $\quad \underline{\frac{1}{16}}$

8. $\frac{5}{9}$ $\quad \frac{5}{12}$ $\quad \frac{2}{3}$ $\quad \frac{5}{6}$ $\quad \frac{7}{8}$ $\quad \frac{3}{8}$ $\quad \frac{5}{6}$ $\quad \frac{7}{8}$

$\underline{\frac{1}{3}}$ $\quad \underline{\frac{1}{3}}$ $\quad \underline{\frac{5}{12}}$ $\quad \underline{\frac{1}{2}}$ $\quad \underline{\frac{3}{4}}$ $\quad \underline{\frac{1}{4}}$ $\quad \underline{\frac{2}{3}}$ $\quad \underline{\frac{1}{2}}$

Subtracting Mixed Numbers

RELATED FRACTIONS; NO REGROUPING

$$2\tfrac{3}{4} = 2\tfrac{3}{4}$$
$$-1\tfrac{1}{2} = 1\tfrac{2}{4}$$
$$\overline{\qquad\quad} \qquad \overline{1\tfrac{1}{4}}$$

1. Ruth wants to subtract $1\frac{1}{2}$ from $2\frac{3}{4}$. She knows that she can subtract only like fractions. What must she do before she can subtract? Can a fraction always be changed to higher terms? Can a fraction always be changed to lower terms? Which of these fractions should Ruth change? The example at the left shows how Ruth did her work. Explain it.

$$8\tfrac{5}{6} = 8\tfrac{5}{6}$$
$$-4\tfrac{1}{2} = 4\tfrac{3}{6}$$
$$\overline{\qquad\quad} \qquad \overline{4\tfrac{2}{6} = 4\tfrac{1}{3}}$$

2. In subtracting $4\frac{1}{2}$ from $8\frac{5}{6}$ Ruth knows that she must change halves to sixths. Why?

   Study her example. Why did she change the $\frac{2}{6}$ in the answer to $\frac{1}{3}$? What is the final answer?

3. Study the examples below. Tell how Ruth found her answers. Copy the examples without the work. Subtract. Compare your answers with those in the book.

$28\frac{7}{8} = 28\frac{7}{8}$ $\quad$ $9\frac{5}{6} = 9\frac{5}{6}$ $\quad$ $5\frac{11}{12} = 5\frac{11}{12}$

$2\frac{1}{2} = 2\frac{4}{8}$ $\quad$ $3\frac{1}{3} = 3\frac{2}{6}$ $\quad$ $3\frac{1}{4} = 3\frac{3}{12}$

$26\frac{3}{8}$ $\quad$ $6\frac{3}{6} = 6\frac{1}{2}$ $\quad$ $2\frac{8}{12} = 2\frac{2}{3}$

Copy and subtract. Be sure all fractions in your answers are in lowest terms.

4.	$8\frac{5}{8}$	$9\frac{3}{5}$	$15\frac{7}{16}$	$27\frac{5}{6}$	$35\frac{7}{10}$	$5\frac{3}{4}$
	$3\frac{1}{2}$	$6\frac{3}{10}$	$2\frac{1}{8}$	$12\frac{1}{12}$	$25\frac{1}{5}$	$2\frac{1}{8}$
5.	$2\frac{1}{3}$	$4\frac{5}{6}$	$7\frac{3}{4}$	$5\frac{5}{6}$	$4\frac{3}{4}$	$3\frac{9}{10}$
	$1\frac{1}{6}$	$3\frac{1}{3}$	$6\frac{5}{12}$	$4\frac{1}{12}$	$4\frac{7}{12}$	$2\frac{3}{5}$
6.	$4\frac{1}{2}$	$6\frac{5}{6}$	$8\frac{2}{3}$	$7\frac{5}{6}$	$6\frac{1}{5}$	$4\frac{8}{9}$
	$1\frac{1}{4}$	$2\frac{2}{3}$	$4\frac{1}{12}$	$4\frac{1}{2}$	$5\frac{1}{10}$	$1\frac{1}{3}$

Subtracting Mixed Numbers

RELATED FRACTIONS WITH REGROUPING

A week ago, Elizabeth had $3\frac{1}{4}$ cans of cat food for her pet cat. She has $1\frac{1}{2}$ cans left. How much food has the cat eaten in the past week?

To find the answer, subtract. Change $\frac{1}{4}$ and $\frac{1}{2}$ to like fractions. $1\frac{1}{2}$ becomes $1\frac{2}{4}$. But you cannot subtract $\frac{2}{4}$ from $\frac{1}{4}$. You know that 3 wholes equal $2\frac{4}{4}$; so $3\frac{1}{4} = 2\frac{4}{4} + \frac{1}{4} = 2\frac{5}{4}$. Now subtract $\frac{2}{4}$ from $\frac{5}{4}$. Then subtract the whole numbers.

$$\begin{array}{r} 3\frac{1}{4} = 3\frac{1}{4} = 2\frac{5}{4} \\ \underline{1\frac{1}{2} = 1\frac{2}{4} = 1\frac{2}{4}} \\ 1\frac{3}{4} \end{array}$$

The cat has eaten $1\frac{3}{4}$ cans of food in the past week.

1. Study these examples. Then copy them without the work. Subtract. Compare your answers with those in the book.

$$\begin{array}{r} 3 = 2\frac{8}{8} \\ \underline{2\frac{5}{8} = 2\frac{5}{8}} \\ \frac{3}{8} \end{array} \qquad \begin{array}{r} 4\frac{3}{4} = 4\frac{6}{8} = 3\frac{14}{8} \\ \underline{3\frac{7}{8} = 3\frac{7}{8} = 3\frac{7}{8}} \\ \frac{7}{8} \end{array} \qquad \begin{array}{r} 5\frac{1}{3} = 5\frac{2}{6} = 4\frac{8}{6} \\ \underline{2\frac{5}{6} = 2\frac{5}{6} = 2\frac{5}{6}} \\ 2\frac{3}{6} = 2\frac{1}{2} \end{array}$$

Copy and subtract. Be sure that fractions in your answers are in lowest terms.

2.	$10\frac{1}{4}$	$9\frac{1}{3}$	6	$7\frac{1}{8}$	$2\frac{1}{12}$	$4\frac{1}{5}$
	$\underline{7\frac{3}{4}}$	$\underline{6\frac{2}{3}}$	$\underline{5\frac{1}{5}}$	$\underline{6\frac{3}{8}}$	$\underline{1\frac{5}{12}}$	$\underline{3\frac{4}{5}}$
3.	3	14	15	8	13	4
	$\underline{2\frac{5}{8}}$	$\underline{8\frac{1}{10}}$	$\underline{7\frac{2}{5}}$	$\underline{7\frac{3}{10}}$	$\underline{2\frac{9}{16}}$	$\underline{2\frac{4}{5}}$
4.	$13\frac{3}{5}$	$11\frac{7}{12}$	$15\frac{1}{12}$	$14\frac{5}{12}$	$12\frac{1}{3}$	$48\frac{3}{10}$
	$\underline{6\frac{9}{10}}$	$\underline{7\frac{3}{4}}$	$\underline{8\frac{5}{6}}$	$\underline{9\frac{3}{4}}$	$\underline{7\frac{5}{6}}$	$\underline{1\frac{2}{5}}$
5.	$15\frac{3}{8}$	$11\frac{5}{8}$	$14\frac{3}{10}$	$9\frac{1}{3}$	$12\frac{1}{2}$	$20\frac{1}{10}$
	$\underline{2\frac{1}{2}}$	$\underline{3\frac{3}{4}}$	$\underline{8\frac{4}{5}}$	$\underline{6\frac{5}{6}}$	$\underline{7\frac{7}{8}}$	$\underline{8\frac{3}{5}}$

Finding Common Denominators

UNRELATED FRACTIONS

Fractions cannot be added or subtracted until they have the same denominator. To take $\frac{1}{2}$ from $\frac{5}{6}$, you first change $\frac{1}{2}$ to $\frac{3}{6}$ so that you can take sixths from sixths. To add $\frac{3}{5}$ and $\frac{7}{10}$, you first change $\frac{3}{5}$ to $\frac{6}{10}$ so you can add tenths to tenths. You say that fractions which you add or subtract must have a ***common denominator.***

1. What common denominator would you use in these examples?

$\frac{5}{8}+\frac{1}{2}$ $\quad$ $\frac{5}{12}-\frac{1}{3}$ $\quad$ $\frac{7}{8}-\frac{3}{4}$ $\quad$ $\frac{1}{10}+\frac{1}{5}$ $\quad$ $\frac{1}{3}+\frac{5}{6}$

In each pair of fractions above, the common denominator is the larger denominator. You would need to change only the fraction with the smaller denominator to have like fractions.

2. To add unlike fractions such as $\frac{1}{2}$ and $\frac{1}{3}$, you must have a common denominator. Can you use 2 for the common denominator? Can you use 3? Neither denominator of the pair will do.
You can change $\frac{1}{2}$ to $\frac{?}{4}$, to $\frac{?}{6}$, to $\frac{?}{8}$, to $\frac{?}{10}$, to $\frac{?}{12}$, to $\frac{?}{16}$, and so on. You can change $\frac{1}{3}$ to $\frac{?}{6}$, to $\frac{?}{9}$, to $\frac{?}{12}$, and so on.
Can you change both $\frac{1}{2}$ and $\frac{1}{3}$ to 4ths? to 6ths? to 8ths? to 10ths? to 12ths? Both 6 and 12 are common denominators for $\frac{1}{2}$ and $\frac{1}{3}$. Are there other larger numbers that could also be used?

$$\begin{array}{r} \frac{1}{2}=\frac{3}{6} \\ +\frac{1}{3}=\frac{2}{6} \\ \hline \frac{5}{6} \end{array}$$

$$\begin{array}{r} \frac{1}{2}=\frac{6}{12} \\ +\frac{1}{3}=\frac{4}{12} \\ \hline \frac{10}{12}=\frac{5}{6} \end{array}$$

Add $\frac{1}{2}$ and $\frac{1}{3}$ using 6 for a common denominator. Then add $\frac{1}{2}$ and $\frac{1}{3}$ using 12 for a common denominator. Compare the two examples. Why may it be better to use the smallest common denominator? The smallest common denominator is often called the ***least common denominator.***

A common denominator is always a number that each of the denominators you are using will divide evenly. The least common denominator is the smallest number for which this is true.

3. What is the smallest number that can be divided evenly by

3 and 4?	4 and 6?	5 and 3?	2 and 5?	4 and 5?
3 and 6?	3 and 7?	8 and 3?	2, 4, and 8?	2, 3, and 4?

Tell what the least common denominator is for each example below. Then work the examples. Watch the signs.

4.

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3}$
$+\frac{1}{6}$	$+\frac{1}{7}$	$+\frac{1}{6}$	$-\frac{1}{4}$	$-\frac{1}{5}$	$-\frac{1}{8}$

5. How did you find the common denominators in row 4?

Mary said, "I found out that if I multiply the denominators of the fractions in an example, their product will always be a common denominator for those fractions. It is not always the least common denominator, but it will always work."

John said, "I start with the largest denominator and see if the other denominators will divide it exactly. If not, I multiply the largest denominator by 2, then by 3, and so on until I come to a number I can use for a common denominator."

Learn to use John's way whenever you cannot tell the least common denominator at a glance.

Copy the following examples. Add or subtract as the signs tell you to do. Be sure the fractions in your answers are proper fractions in lowest terms.

6.

$4\frac{2}{3}$	$2\frac{3}{4}$	$5\frac{3}{4}$	$4\frac{5}{6}$	$5\frac{1}{2}$	$2\frac{3}{4}$
$+3\frac{1}{4}$	$+3\frac{1}{6}$	$+2\frac{1}{5}$	$+3\frac{1}{12}$	$+4\frac{2}{3}$	$+6\frac{3}{5}$

7.

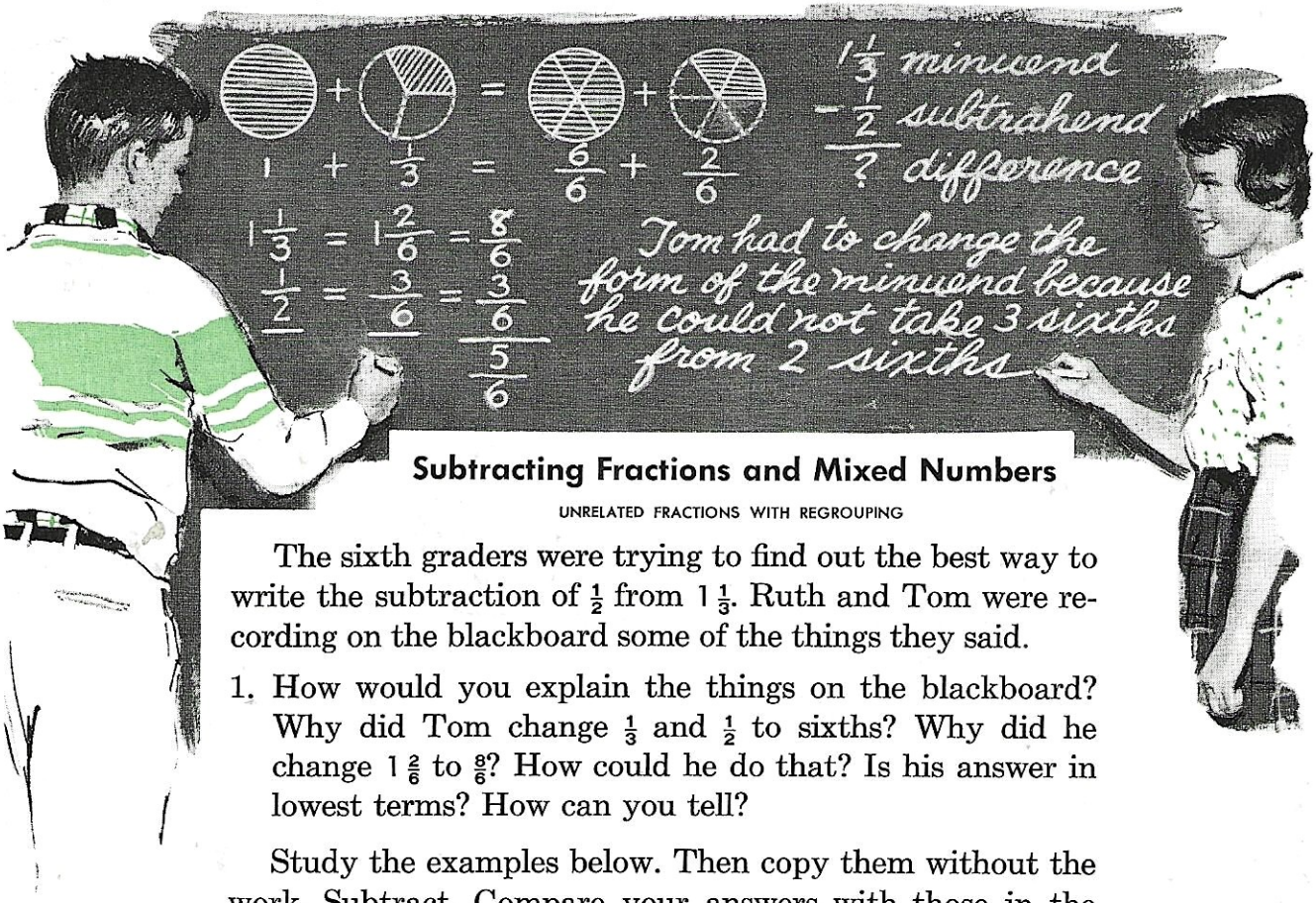
$2\frac{2}{5}$	$4\frac{3}{4}$	$5\frac{2}{3}$	$3\frac{5}{8}$	$8\frac{1}{3}$	$9\frac{3}{5}$
$-1\frac{1}{10}$	$-2\frac{2}{3}$	$-1\frac{1}{2}$	$-3\frac{1}{2}$	$-4\frac{1}{4}$	$-6\frac{1}{3}$

8.

$\frac{3}{8}$	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$
$+\frac{1}{2}$	$+\frac{5}{12}$	$+\frac{5}{12}$	$+\frac{1}{4}$	$+\frac{3}{4}$	$+\frac{1}{5}$

9.

$\frac{5}{6}$	$9\frac{2}{3}$	$9\frac{4}{5}$	$4\frac{2}{3}$	$10\frac{3}{4}$	$8\frac{1}{4}$
$-\frac{3}{4}$	$-8\frac{1}{4}$	$-1\frac{2}{3}$	$-2\frac{1}{2}$	$-7\frac{1}{3}$	$-2\frac{1}{6}$


1 + 1/3 = 6/6 + 2/6
1 1/3 = 1 2/6 = 8/6
1/2 = 3/6 = 3/6
5/6
1 1/3 minuend
- 1/2 subtrahend
? difference
Tom had to change the form of the minuend because he could not take 3 sixths from 2 sixths

Subtracting Fractions and Mixed Numbers

UNRELATED FRACTIONS WITH REGROUPING

The sixth graders were trying to find out the best way to write the subtraction of $\frac{1}{2}$ from $1\frac{1}{3}$. Ruth and Tom were recording on the blackboard some of the things they said.

1. How would you explain the things on the blackboard? Why did Tom change $\frac{1}{3}$ and $\frac{1}{2}$ to sixths? Why did he change $1\frac{2}{6}$ to $\frac{8}{6}$? How could he do that? Is his answer in lowest terms? How can you tell?

Study the examples below. Then copy them without the work. Subtract. Compare your answers with those in the book.

2. $6\frac{1}{2} = 6\frac{3}{6} = 5\frac{9}{6}$
$\underline{5\frac{2}{3} = 5\frac{4}{6} = 5\frac{4}{6}}$
$\frac{5}{6}$

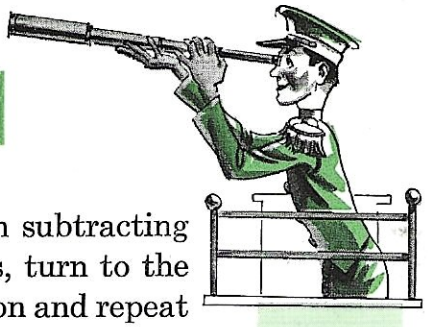
$9\frac{1}{5} = 9\frac{2}{10} = 8\frac{12}{10}$
$\underline{4\frac{7}{10} = 4\frac{7}{10} = 4\frac{7}{10}}$
$4\frac{5}{10} = 4\frac{1}{2}$

Copy and subtract. Be sure the fractions in your answers are in lowest terms.

3. $9\frac{3}{4}$	$14\frac{1}{2}$	$16\frac{1}{3}$	$10\frac{2}{5}$	$9\frac{1}{2}$	$20\frac{1}{8}$
$\underline{4\frac{2}{3}}$	$\underline{7\frac{3}{5}}$	$\underline{\frac{1}{2}}$	$\underline{9\frac{3}{4}}$	$\underline{3\frac{3}{5}}$	$\underline{8\frac{2}{3}}$
4. $2\frac{3}{8}$	$7\frac{1}{4}$	$8\frac{1}{2}$	$15\frac{2}{5}$	$12\frac{1}{2}$	$17\frac{1}{9}$
$\underline{\frac{5}{6}}$	$\underline{6\frac{1}{3}}$	$\underline{\frac{4}{5}}$	$\underline{3\frac{2}{3}}$	$\underline{6\frac{2}{3}}$	$\underline{\frac{1}{2}}$

Checking Up on Fractions and Mixed Numbers

DIAGNOSTIC TEST ON PP. 111–116



This is a test to find out if you need more help in subtracting fractions. If you make mistakes in a row of examples, turn to the page shown at the end of the row. Study the explanation and repeat the practice. Then try the row on this page again.

Subtract. Remember to change proper fractions in the answers to lowest terms.

1.	$\frac{9}{10}$	$\frac{1}{3}$	$\frac{7}{8}$	$\frac{2}{3}$	$\frac{4}{7}$	$\frac{15}{16}$	111
	$-\frac{2}{5}$	$-\frac{1}{6}$	$-\frac{1}{4}$	$-\frac{1}{12}$	$-\frac{3}{14}$	$-\frac{5}{8}$	
2.	$3\frac{1}{4}$	$7\frac{1}{2}$	$4\frac{5}{12}$	$6\frac{3}{5}$	$8\frac{11}{14}$	$6\frac{1}{3}$	112
	$-1\frac{1}{8}$	$-2\frac{1}{6}$	$-1\frac{1}{3}$	$-2\frac{4}{15}$	$-1\frac{3}{7}$	$-5\frac{2}{9}$	
3.	$4\frac{1}{8}$	$8\frac{1}{6}$	$9\frac{1}{2}$	$8\frac{3}{16}$	$6\frac{1}{4}$	$6\frac{3}{10}$	113
	$-1\frac{1}{4}$	$-4\frac{2}{3}$	$-6\frac{7}{12}$	$-4\frac{3}{4}$	$-2\frac{7}{8}$	$-3\frac{4}{5}$	
4.	$\frac{2}{3}$	$\frac{1}{2}$	$5\frac{2}{3}$	$7\frac{1}{4}$	$8\frac{3}{5}$	$12\frac{2}{3}$	115
	$-\frac{1}{4}$	$-\frac{2}{5}$	$-3\frac{1}{2}$	$-3\frac{1}{6}$	$-5\frac{1}{3}$	$-7\frac{1}{2}$	
5.	$4\frac{1}{8}$	$8\frac{1}{6}$	$14\frac{1}{3}$	$9\frac{2}{5}$	$15\frac{1}{9}$	$10\frac{1}{2}$	116
	$-2\frac{3}{4}$	$-4\frac{2}{3}$	$-\frac{1}{2}$	$-5\frac{3}{4}$	$-\frac{1}{2}$	$-7\frac{3}{5}$	

Copy and add. Change improper fractions in the answers to whole numbers or mixed numbers. Be sure proper fractions are in lowest terms.

6.	$5\frac{1}{4}$	$6\frac{1}{6}$	$5\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	115
	$+2\frac{2}{3}$	$+4\frac{3}{4}$	$+6\frac{1}{5}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{1}{4}$	
				$+\frac{1}{2}$	$+\frac{3}{4}$	$+\frac{4}{5}$	

What Have You Learned about Fractions?

CHAPTER TEST ON MEANINGS AND SKILLS

1. In the fraction $\frac{3}{8}$, the numerator is _?_; the denominator is _?_.
2. A fraction is in lowest terms when no whole number except _?_ will divide both the numerator and denominator evenly.
3. Write the following fractions in lowest terms: $\frac{6}{8}$ $\frac{8}{20}$ $\frac{9}{24}$
4. Write the following names on your paper one under the other: *proper fraction, improper fraction, whole number, mixed number.* Give two examples after each name.
5. Write a fraction in lowest terms to compare 5″ with 10″.

Copy and add. Be sure that all fractions in the sums are proper fractions in lowest terms.

6.	$\frac{2}{7}$	$\frac{5}{8}$	$\frac{2}{3}$	$17\frac{1}{4}$	$8\frac{3}{8}$	$11\frac{10}{12}$
	$\frac{3}{7}$	$\frac{3}{8}$	$\frac{2}{3}$	$\frac{3}{4}$	$4\frac{7}{8}$	$6\frac{6}{12}$
7.	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	$4\frac{1}{2}$	$15\frac{7}{8}$	$12\frac{1}{4}$
	$\frac{1}{6}$	$\frac{5}{8}$	$\frac{1}{4}$	$2\frac{1}{4}$	$\frac{1}{2}$	$6\frac{3}{5}$
8.	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{9}$	$8\frac{5}{12}$	$18\frac{1}{3}$	$9\frac{1}{2}$
	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{1}{3}$	$6\frac{1}{2}$	$7\frac{2}{9}$	$3\frac{3}{4}$
	$\frac{5}{8}$	$\frac{2}{4}$	$\frac{7}{9}$	$4\frac{7}{12}$	$4\frac{2}{3}$	$5\frac{4}{5}$

Copy and subtract. Change answers to lowest terms.

9.	$\frac{4}{5}$	$\frac{5}{8}$	$\frac{5}{6}$	$\frac{7}{10}$	$\frac{1}{2}$	$\frac{3}{4}$
	$\frac{1}{5}$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$
10.	$4\frac{1}{2}$	$15\frac{7}{8}$	$8\frac{5}{6}$	$34\frac{7}{10}$	$23\frac{5}{6}$	$30\frac{5}{8}$
	$2\frac{1}{4}$	$\frac{1}{2}$	$4\frac{1}{3}$	$21\frac{1}{2}$	$12\frac{5}{12}$	$15\frac{7}{16}$

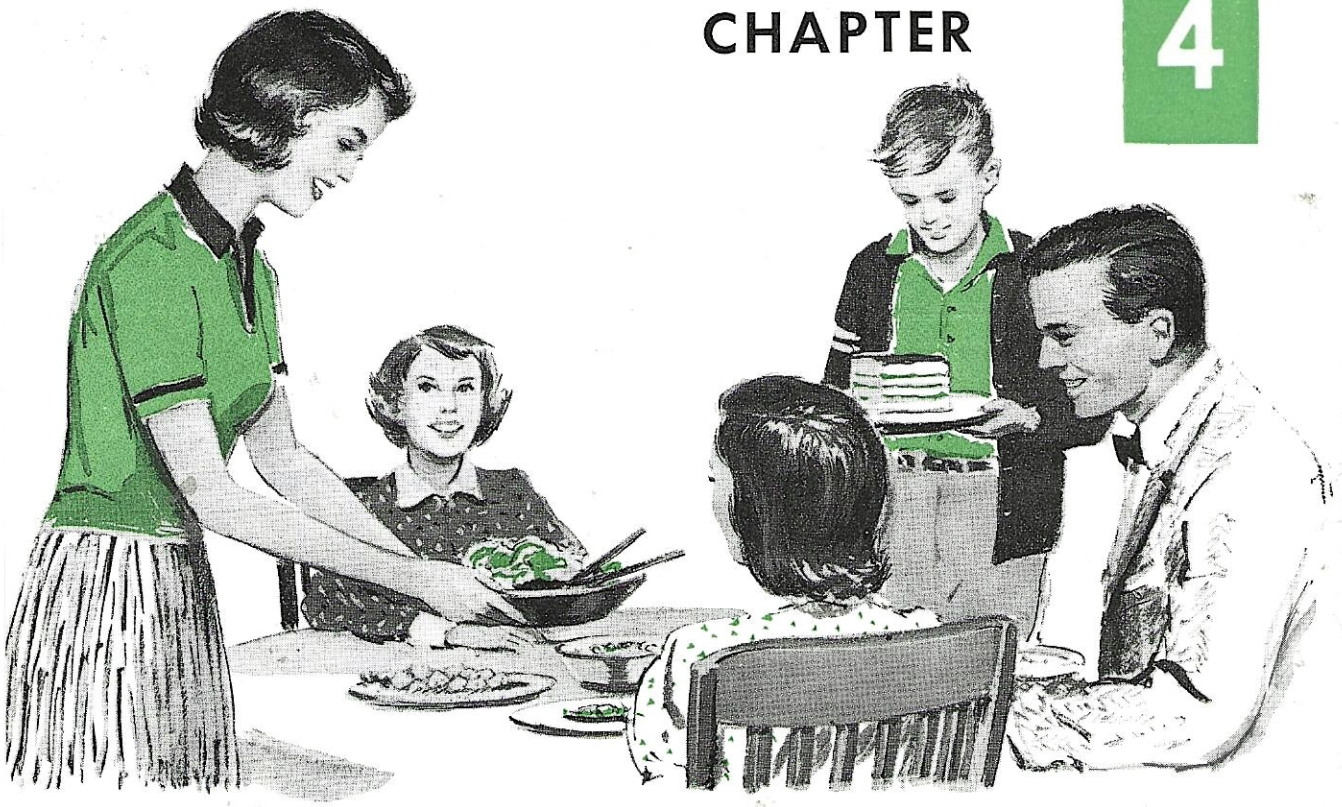
11. $7\frac{3}{8}$	1	5	12	$8\frac{1}{16}$	11
5	$\frac{1}{3}$	$\frac{3}{4}$	$5\frac{4}{12}$	$\frac{9}{16}$	$4\frac{2}{6}$
12. $9\frac{1}{6}$	$7\frac{1}{8}$	$6\frac{1}{3}$	$5\frac{1}{4}$	$11\frac{3}{10}$	$15\frac{1}{3}$
$2\frac{5}{6}$	$\frac{3}{4}$	$3\frac{5}{6}$	$2\frac{1}{3}$	$6\frac{4}{5}$	$5\frac{1}{2}$

What Have You Learned about Fractions?

CHAPTER TEST ON PROBLEM SOLVING

1. At 80¢ a pound, how much does $\frac{3}{4}$ of a pound of chopped sirloin steak cost?
2. It is $1\frac{7}{10}$ miles from Fred's house to the park. What is the distance from Fred's house to the park and back?
3. This year Norman measures $58\frac{3}{8}$ inches tall. Last year he measured $56\frac{1}{4}$ inches. How much has he grown?
4. The Boy Scouts are selling fruit cakes. One cake weighs 3 pounds; another cake weighs $1\frac{3}{4}$ pounds. How much more does the larger cake weigh?
5. Nancy is making an apron. When finished, it will be 14 inches long. Allowing $2\frac{1}{2}$ inches for a hem and $\frac{3}{4}$ of an inch for gathering at the top, how long a piece of muslin will Nancy need for the apron?
6. Mrs. Brady bought two broilers. One weighed $3\frac{1}{4}$ pounds; the other weighed $2\frac{3}{4}$ pounds. What was the difference in the weights of these broilers?
7. Billie and Anne are twins. When they were born, Billie weighed $4\frac{1}{2}$ pounds and Anne weighed $4\frac{3}{16}$ pounds. How much more did Billie weigh than Anne?
8. Jack's pet cat, Davy Crockett, weighs $2\frac{1}{2}$ pounds. When Jack got him, he weighed $\frac{3}{4}$ of a pound. How much weight has Davy Crockett gained?

CHAPTER 4



How Many and How Much?

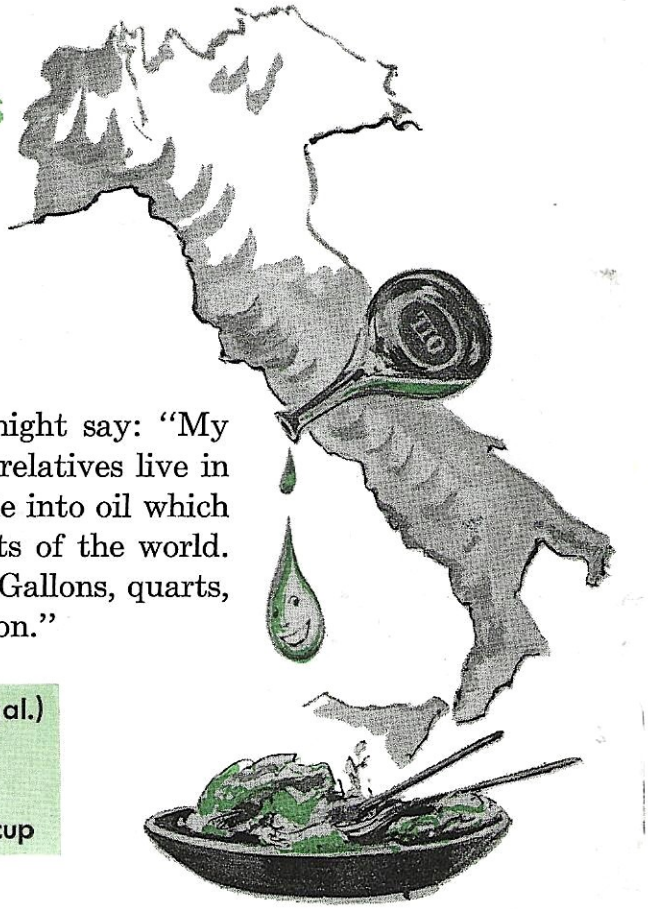
Have you ever thought how often we have to answer the questions how many and how much?

Everything we buy and almost everything we use has been counted or measured in some way. Often one article has been measured in several ways before it is ready for us to use. For example, apples might have been graded according to size (linear measure), packed in bushel baskets (volume), and then sold by the pound (weight).

Suppose that instead of the people in the picture talking, the food and the other materials in the dining room tell about their experiences with measurement.

See if you can make up the stories that some of them might tell. You may like your own stories better than those which follow in the book.

Measures and Amounts



Liquid Measure

GALLON, QUART, PINT, CUP, FLUID OUNCE

The olive oil in the salad dressing might say: "My family tree is in Italy, but some of my relatives live in the United States. Many olives are made into oil which is canned or bottled and sent to all parts of the world. The containers used are of many sizes. Gallons, quarts, pints, and half pints are the most common."

4 quarts (qt.) = 1 gallon (gal.)
2 pints (pt.) = 1 quart
2 cups (c.) = 1 pint
8 fluid ounces (fl. oz.) = 1 cup

1. Why is 1 cup equal to 1 half pint?
2. How many fluid ounces make 1 cup? $\frac{1}{2}$ cup? 3 cups?
3. How many quarts are there in a 2-gallon can of olive oil?
4. How many pints in a gallon? In 2 gallons? In $\frac{1}{2}$ gallon?
5. Answer the following questions; then say how you can tell whether to multiply or divide when you are changing one unit of measure into another unit.

   In any given amount of olive oil, will there be
   (a) More quarts than gallons?
   (b) Fewer quarts than pints?
   (c) More cups than pints?
   (d) More fluid ounces than cups?

Now turn the page.

Copy and fill in the blanks. Never write in your book.

6. 2 pt. = ? c. 4 qt. = ? pt. 4 gal. = ? qt.
7. 3 gal. = ? pt. 3 qt. = ? c. 1 pt. = ? fl. oz.
8. 8 c. = ? pt. 12 pt. = ? qt. 24 qt. = ? gal.
9. 2 qt. = ? gal. 3 qt. = ? gal. 16 c. = ? gal.
10. 3 gal. 1 qt. = ? qt. 1 pt. 8 fl. oz. = ? pt.

Linear Measure

INCH, FOOT, YARD, ROD, MILE

The tablecloth might say: "I come from the flax fields in Ireland. Some flax fields are miles long. After being cut, flax fiber is made into linen, lace, cord, and rope. I was woven into damask 2 yards wide, and then cut into lengths of from 4 feet to 24 feet. After a $\frac{1}{2}$-inch hem was made at the ends of each piece, fine tablecloths were ready for the markets of the world."

12	inches (in.)	= 1	foot (ft.)
3	feet	= 1	yard (yd.)
36	inches	= 1	yard
$5\frac{1}{2}$	yards	= 1	rod (rd.)
$16\frac{1}{2}$	feet	= 1	rod
5280	feet	= 1	mile (mi.)

1. Find *line* in the word *linear*. When you measure the length of a tablecloth, can you say that you are measuring along a line? When do you use linear measure?

2. What unit of linear measure might be used to measure your height? The length of a board to build an outdoor table? The length of a rug for a small room? The width of a piece of paper? The length of cloth needed for a long dress? The distance between two cities? The length of a field?

Copy and fill in the blanks. Use the idea which you developed on page 121 to decide whether to multiply or divide.

3. 48 in. = _?_ ft. 12 ft. = _?_ yd. 72 in. = _?_ yd.

4. 11 yd. = _?_ ft. 2 rd. = _?_ ft. 2 rd. = _?_ yd.

5. 2 mi. = _?_ ft. 26,400 ft. = _?_ mi. $\frac{1}{2}$ mi. = _?_ ft.

6. 5 ft. 11 in. = _?_ in. 65 in. = _?_ ft. _?_ in.

7. 3 yd. 7 in. = _?_ in. 88 in. = _?_ yd. _?_ in.

8. 11 yd. 2 ft. = _?_ ft. 16 ft. = _?_ yd. _?_ ft.

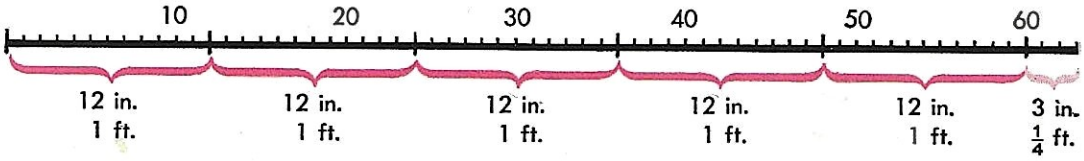
9. John said that the width of his father's house is 2 rods. How many feet wide is the house? How many yards?

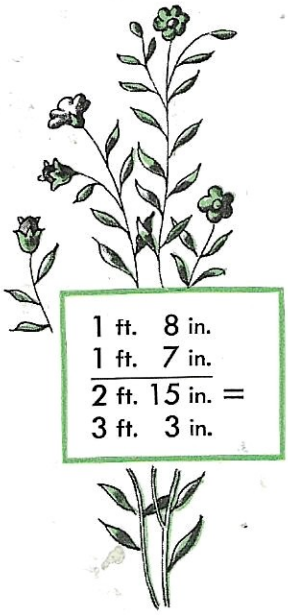
10. Mount Everest, the highest mountain peak in the world, is about 29,000 feet high. Is it more than 5 miles high? In what two ways can you find the answer? Which way is easier?

11. Marie needs 18 inches of material to make a doll's dress. What fraction of a yard should she ask for?

12. Joe is $5\frac{1}{2}$ feet tall. What is his height in inches?

13. Phil is 63 inches tall. Express his height in feet and inches. Express his height in feet. Use the drawing below if you need help.





1 ft. 8 in.
1 ft. 7 in.
2 ft. 15 in. =
3 ft. 3 in.

Adding Measures

At the end of June a flax plant was 1 foot 8 inches tall. During July and August it grew 1 foot 7 inches more. How tall was it then?

Why should you add to find the answer?

Study the example at the left. Is 15 in. more than 1 ft.? How much more? By regrouping, can you change 15 in. to 1 ft. 3 in. and carry 1 ft. to the column at the left? Why is the answer 3 ft. 3 in., not 3 ft. 5 in.?

Copy and complete.

1.

2 ft. 9 in.	3 yd. $2\frac{1}{2}$ ft.	5 gal. 3 qt.
4 ft. 6 in.	1 yd. $2\frac{1}{2}$ ft.	2 gal. 3 qt.
6 ft. 15 in. =	4 yd. 5 ft. =	7 gal. 6 qt. =
7 ft. ? in.	5 yd. ? ft.	? gal. ? qt.

2. Think about the examples you have just completed. In each of them, did you regroup the numbers in the answer? Is this similar to the regrouping you do in other additions when you have a number to carry? In adding measures do you sometimes have to regroup when there is less than 10 in the right-hand column of the sum? Why? How can you tell how to regroup when adding numbers obtained by making a measurement?

Copy and add. Think what you are doing. Perhaps you can do the regrouping mentally.

3.

5 ft. 10 in.	7 gal. 3 qt.	8 yd. 20 in.
3 ft. 9 in.	4 gal. 3 qt.	10 yd. 24 in.

4.

2 qt. 1 pt.	6 yd. $2\frac{1}{2}$ ft.	15 gal. $3\frac{1}{2}$ qt.
1 qt. $1\frac{1}{2}$ pt.	2 yd. $1\frac{1}{2}$ ft.	12 gal. $2\frac{1}{2}$ qt.

5.

1 yd. 2 ft.	6 yd. 20 in.	6 qt. 1 pt.
2 yd. 2 ft.	4 yd. 24 in.	2 qt. 1 pt.
3 yd. 1 ft.	7 yd. 11 in.	3 qt. 1 pt.

Subtracting Measures

The dining-room table at the Browns' is 5 ft. 4 in. long. The breakfast table is 3 ft. 9 in. long. How much longer is the dining-room table than the breakfast table?

Why should you subtract to find the answer?

Study the example at the right. Can you subtract 9 in. from 4 in.? How can you get more inches to subtract from? After you change 1 of the 5 ft. to 12 in., what numbers do you have in the minuend? Think about the subtraction. Answer the question the problem asks.

4	16
5 ft.	4 in.
3 ft.	9 in.
1 ft.	7 in.

Copy and complete.

7 4	2 6	5 36 in.
1. 8 yd. 1 ft.	3 gal. 2 qt.	6 yd.
2 yd. 2 ft.	2 gal. 3 qt.	1 yd. 17 in.

2. Think about the examples you have just completed. In each of them, did you have to regroup the numbers in the minuend? Why? Is this similiar to the regrouping you do in other subtraction examples when you have to borrow? Why is 8 yd. 1 ft. not equal to 7 yd. 11 ft.? Why is 3 gal. 2 qt. not equal to 2 gal. 12 qt.? Why is 6 yd. not equal to 5 yd. 10 in.? How can you tell how much to "borrow" when subtracting measurement numbers?

Copy and subtract. Think what you are doing. Perhaps you can do the regrouping mentally.

3.	7 yd. 2 ft. 4 yd. 1 ft.	9 yd. 1 ft. 6 yd. 2 ft.	9 gal. 2 qt. 4 gal. 1 qt.
4.	9 gal. 1 qt. 3 gal. 3 qt.	5 ft. 6 in. 4 ft. 10 in.	15 gal. 2 qt. 11 gal. $3\frac{1}{2}$ qt.
5.	8 yd. 4 yd. 2 ft.	8 qt. 1 pt. 5 qt. $2\frac{1}{2}$ pt.	12 ft. 4 in. 8 ft. 9 in.

Small Measures Used in Cooking

TEASPOON, TABLESPOON, FLUID OUNCE, CUP

The vanilla in the cake might say: "The vanilla beans from which I was made are among the best-known products of Madagascar, an island halfway around the world. The beans grow on a climbing plant in the hills and are picked before they are ripe. Then the oil is taken from the beans and manufactured into vanilla extract, the liquid that most people call vanilla. Vanilla is packed in bottles and shipped around the world. It is used in small amounts. The bottles are usually labeled in fluid ounces. Mrs. Brown measures vanilla with a teaspoon."

The teaspoon, tablespoon, and cup are used both for liquids and for dry ingredients like sugar or baking powder.

3 teaspoons (tsp.) = 1 tablespoon (tbs.)
2 tablespoons = 1 fluid ounce (fl. oz.)
16 tablespoons = 1 cup (c.)

Look carefully at the abbreviations for teaspoon and tablespoon. Notice that *tablespoon* has a *b* in it. Then it will be easy to remember which is which.

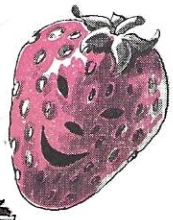
Copy and fill in the blanks.

1. 6 tsp. = _?_ tbs. 6 tbs. = _?_ tsp. 2 c. = _?_ tbs.
2. 1 pt. = _?_ tbs. $\frac{1}{4}$ c. = _?_ tsp. 8 tbs. = _?_ c.
3. Mrs. Brown used 4 tablespoons of sugar to make a sauce. What part of a cup did she use? Suppose she wanted to make twice as much sauce. How much sugar should she use?
4. To make ice cream, 2 teaspoons of vanilla were used. What part of a tablespoon was used?
5. A new bottle contained 4 fl. oz. of vanilla. How many teaspoons of vanilla did the bottle hold?

Dry Measure

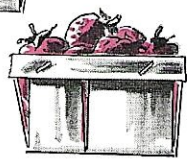
PINT, QUART, PECK, BUSHEL

"It is, indeed, an honor to be at the same table with things that come from so far away," the red-faced strawberries in the bowl might whisper to one another. "Only a few hours ago we were growing in the warm sun on a near-by farm. After we were picked, we were placed in pint and quart boxes, and taken to market. There we saw other fresh fruits and vegetables. Some were packed in peck and bushel baskets."



The measures mentioned by the strawberries are *dry measures*:

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)



Notice that pints and quarts appear in the tables of both liquid measure and dry measure. Both liquid measure and dry measure tell how much space there is in the container. We say they measure *volume* or *capacity*. Liquid quarts and dry quarts are not quite the same size. The dry quart is just a little bigger.

1. Would a dry pint be just a little bigger than a liquid pint?
2. How many quart baskets can be filled from a bushel of beans?

Copy and fill in the blanks.

3. 8 qt. = ? pt. 4 bu. = ? pk. 8 pk. = ? qt.
4. 2 pk. = ? pt. 16 qt. = ? pk. 6 pt. = ? qt.
5. 36 pk. = ? bu. 32 qt. = ? bu. $\frac{3}{4}$ bu. = ? pk.
6. $\frac{1}{8}$ of 32 qt. = ? qt. $\frac{1}{2}$ of 16 qt. = ? pk. $\frac{1}{8}$ pk. = ? qt.

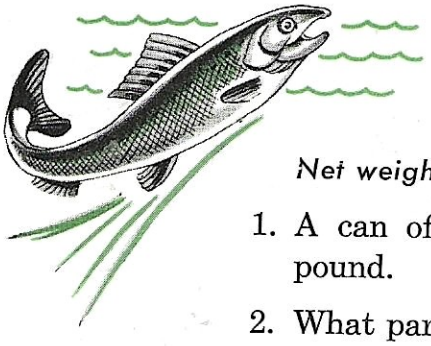
Copy and add. Then copy the same examples again and subtract. Be careful in regrouping the numbers.

7. 3 pk. 5 qt. 5 bu. 2 pk. 6 qt. $\frac{1}{2}$ pt.
   2 pk. 7 qt. 3 bu. 3 pk. 3 qt. $1\frac{1}{2}$ pt.

Measures of Weight

OUNCE, POUND, TON

The salmon's story might be like this: "When I was caught in my native stream in Alaska, I was a gleaming, beautiful fish. I am still proud of my appearance. Tons of salmon go to canneries every year. They are put into cans holding various weights from 4 ounces to 1 pound."



16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

Net weight is the weight of the contents of a container.

1. A can of salmon marked *8 ounces, net weight,* holds $\frac{?}{16}$ or $\frac{?}{?}$ pound.
2. What part of a pound is 2 oz.? 4 oz.? 12 oz.?
3. Mrs. Brown opened a pound can of salmon but used only $\frac{3}{4}$ of it. How many ounces did she use?

Find the answers.

4. $\frac{1}{2}$ of 16 oz. $\frac{1}{8}$ of 16 oz. $\frac{3}{8}$ of 16 oz. $\frac{5}{8}$ of 16 oz.
5. $\frac{1}{2}$ of 4 lb. $\frac{1}{4}$ of 4 lb. $\frac{3}{4}$ of 4 lb. $\frac{2}{3}$ of 12 T.
6. A truck carried 2 tons of salmon to the cannery. How many pounds of salmon was this?

Copy and fill in the blanks.

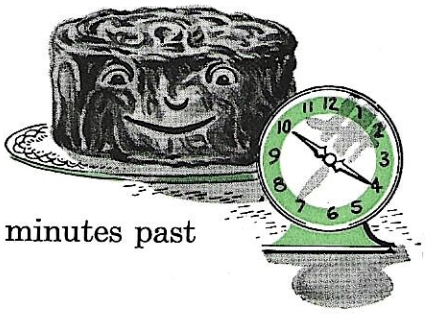
7. 6 lb. = _?_ oz. 4 T. = _?_ lb. 24 oz. = _?_ lb.
8. 6 oz. = _?_ lb. 10 oz. = _?_ lb. 80 oz. = _?_ lb.
9. 6000 lb. = _?_ T. 8000 lb. = _?_ T. 1000 lb. = _?_ T.
10. $\frac{1}{2}$ ft. = _?_ in. $\frac{3}{4}$ ft. = _?_ in. $\frac{1}{3}$ yd. = _?_ in.
11. $\frac{1}{4}$ mi. = _?_ ft. $\frac{3}{4}$ mi. = _?_ ft. $\frac{3}{8}$ mi. = _?_ ft.
12. $\frac{1}{2}$ mi. = _?_ ft. $\frac{3}{4}$ mi. = _?_ yd. 2 rd. = _?_ yd.

Small Units of Time

SECOND, MINUTE, HOUR, DAY

The cake might say: "Time is very important in making an elegant cake like me. After being mixed very carefully with just the right amount of beating, I was baked for 35 minutes. Mrs. Brown let me cool for 1 hour, then covered me with Seven Minute Icing."

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)



1. If Mrs. Brown put the cake into the oven at 20 minutes past 10, at what time should she take it out?

Copy and fill in the blanks.

2. 3 min. = _?_ sec. 4 hr. = _?_ min. 96 hr. = _?_ da.
3. 240 sec. = _?_ min. $\frac{3}{4}$ min. = _?_ sec. $\frac{5}{6}$ hr. = _?_ min.
4. $\frac{3}{8}$ da. = _?_ hr. 20 min. = _?_ hr. 30 sec. = _?_ min.
5. 6 hr. = _?_ da. $\frac{1}{3}$ da. = _?_ min. 180 min. = _?_ da.
6. 2 hr. 15 min. = _?_ min. 3 da. 6 hr. = _?_ hr.
7. 280 min. = _?_ hr. _?_ min. 1440 min. = _?_ da. _?_ hr.

Time between midnight and noon is A.M. and between noon and midnight is P.M. Twelve o'clock noon is sometimes written 12:00 N. Twelve o'clock midnight is sometimes written 12:00 M.

Read and tell the time of day for the following:

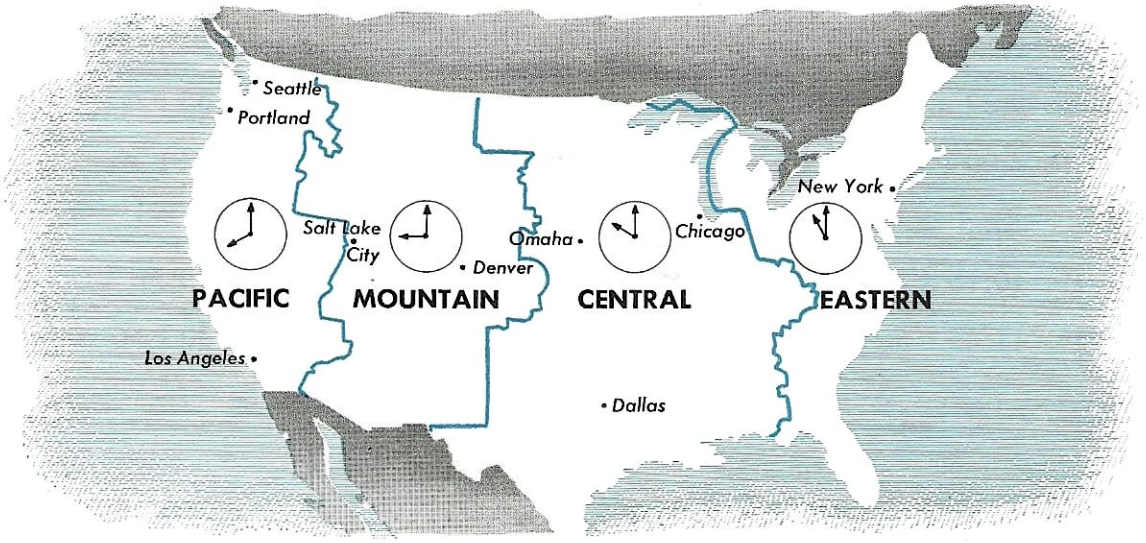
8. 9:32 A.M. 10:21 P.M. 12:00 N. 6:30 A.M. 7:35 P.M.
9. 4:00 P.M. 3:02 A.M. 7:15 A.M. 12:03 A.M. 5:02 P.M.

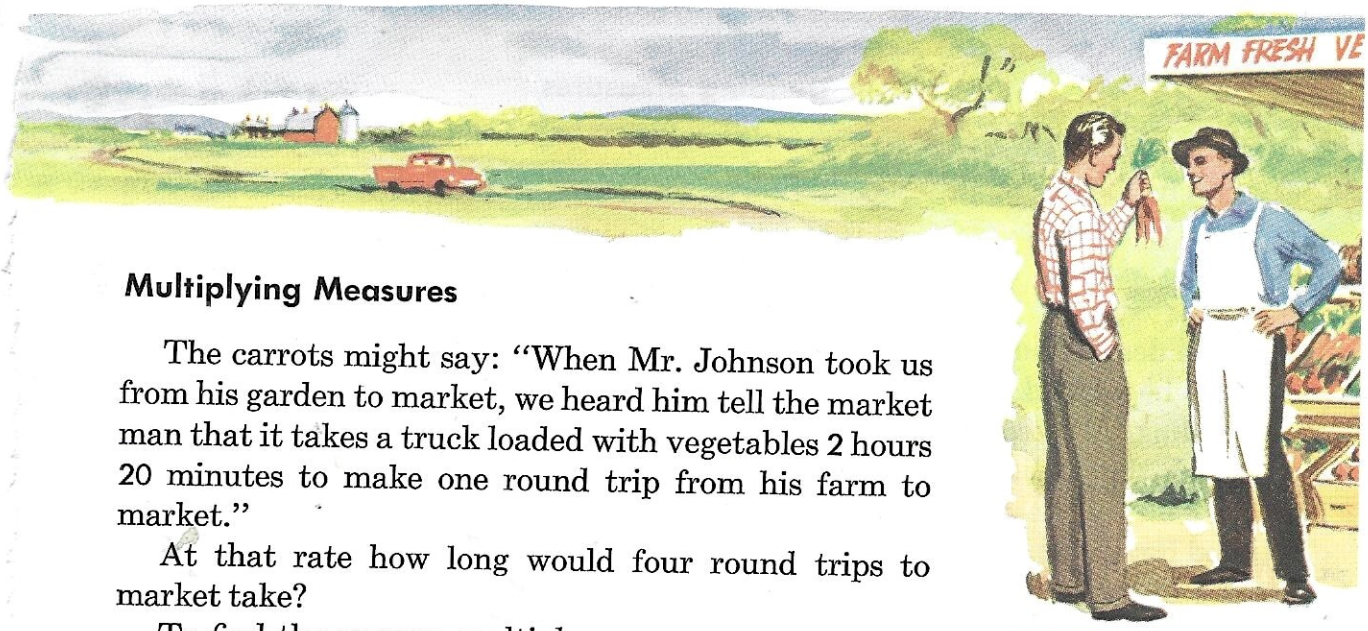
Find the number of minutes between:

10. 10:15 A.M. and 11:45 A.M. 4:35 P.M. and 5:10 P.M.
11. 11:45 P.M. and 1:15 A.M. 10:25 A.M. and 2:30 P.M.

Time Zones

1. At the same time that it is 8:00 A.M. in California, what is the time in Colorado? Illinois? New York?
2. If a fruit grower in Portland, Oregon is working in his apple orchard at 2:00 P.M., what time do clocks show in the Mountain Zone? Central Zone? Eastern Zone?
3. A Massachusetts cranberry grower was working in his cranberry bog at 9:00 A.M. Eastern Time. At that same time what hour did the clocks show in Dallas? Denver? Los Angeles?
4. Mrs. Macy got home from shopping at 1:30 P.M. in New York City. What time was it then in Seattle? In Salt Lake City? In Omaha?
5. A farmer working in his cornfield in Illinois looks at his watch which reads 10:35 A.M. What time would it be in Washington? Wyoming? New Jersey?
6. What should a traveler do to his watch as he crosses the country from west to east? From east to west?
7. A plane leaves Dallas at 4:30 P.M. Central Time and arrives at New York at 10:20 P.M. Eastern Time. How long did the trip take?
8. Look up *Time* or *Standard Time* in your school's encyclopedia and make a report to your class on the history or on the science of measuring time.





Multiplying Measures

The carrots might say: "When Mr. Johnson took us from his garden to market, we heard him tell the market man that it takes a truck loaded with vegetables 2 hours 20 minutes to make one round trip from his farm to market."

At that rate how long would four round trips to market take?

To find the answer, multiply.

First multiply the minutes. 4×20 min. $= 80$ min.

Then multiply the hours. 4×2 hr. $= 8$ hr.

Since 80 minutes are more than 1 hour, change 80 minutes to 1 hour 20 minutes. 8 hr. + 1 hr. 20 min. = 9 hr. 20 min.

Four round trips will take 9 hours 20 minutes.

2 hr. 20 min.
×4
8 hr. 80 min. =
9 hr. 20 min.

1. Study the example at the right. Copy it without answers. Close your book and multiply. Notice that 20 in. is changed to 1 ft. 8 in., then added to 25 ft.

5 ft. 4 in.
×5
25 ft. 20 in. =
26 ft. 8 in.

Copy and multiply.

2. 6 lb. 3 oz. ×7 — 2 ft. 3 in. ×8 — 3 pk. 3 qt. ×5

3. 6 min. 20 sec. ×4 — 8 gal. 3 qt. ×6 — 2 da. 18 hr. ×3

4. A large can of salmon weighs 1 pound 3 ounces. How much will a dozen of these large cans weigh?

5. Mrs. Brown wants to make 6 towels. She needs 1 yard 8 inches of material for each of them. How much material will she need?

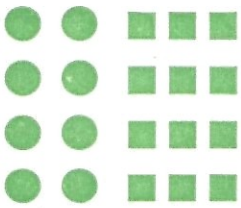
Dividing Measures

PARTITIVE DIVISION

The beans might say: "We are part of a bushel of beans that had a rough ride to market in the same truck with the carrots."

A dealer had 8 lb. 12 oz. of beans that he wanted to put in 4 bags each holding the same amount. How much should he put in each bag?

To find the answer, you have to find how much would be in each part if 8 lb. 12 oz. of beans were divided into 4 equal parts. Find the answer by using the drawing below. Each circle stands for 1 lb., and each square stands for 1 oz.



What does your answer mean?

Is this a problem in division? Study the example below.

$$4\overline{)8 \text{ lb. } 12 \text{ oz.}}\quad 2 \text{ lb. } 3 \text{ oz.}$$

Notice that you can divide both units of measure into 4 equal parts without a remainder.

Find the answers.

1. 12 ft. 6 in. ÷ 6
2. 8 bu. 2 pk. ÷ 2
3. 16 min. 20 sec. ÷ 4
4. 2 lb. 7 oz. ÷ 3

Can you divide 2 lb. by 3? Then change 2 lb. 7 oz. to ounces, and divide. The answer means ___?___.

5. 4 da. 12 hr. ÷ 9
6. 1 gal. 1 qt. ÷ 5
7. 1 lb. 4 oz. ÷ 5
8. 1 mi. 720 ft. ÷ 6

How do you check division examples? Do you think these examples could be checked in the same way?

Look back at your answers for examples 1 and 8. Then study the checks below.

Check for example 1:

$$\begin{array}{r} 2 \text{ ft. } 1 \text{ in.} \\ \times 6 \\ \hline 12 \text{ ft. } 6 \text{ in.} \end{array}$$

Check for example 8:

$$\begin{array}{r} 1000 \text{ ft.} \\ \times 6 \\ \hline 6000 \text{ ft.} = \\ 1 \text{ mi. } 720 \text{ ft.} \end{array}$$

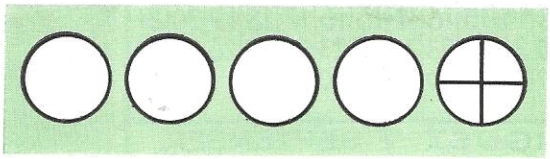
9. Check your answers for examples 2–7.

Dividing Measures

MEASUREMENT DIVISION

If it takes Mr. Johnson 1 hour 15 minutes to pick one bushel of beans, how many bushels can he pick in 5 hours?

To find the answer, you have to find how many groups of 1 hr. 15 min. there are in 5 hr. See if you can find the answer by using the drawing below, letting each circle stand for 1 hour.



What does your answer mean?

Is this a problem in division? Can you divide 5 hr. by 1 hr. 15 min. without changing the form of the numbers? You may change the numbers in two ways. Study the examples below. Explain them. Which way do you like better? The second way is often easier, particularly if there is a remainder when you divide.

$$\begin{array}{r} 4 \\ 1 \text{ hr. } 15 \text{ min.})\overline{4 \text{ hr. } 60 \text{ min.}} \\ \underline{4 \text{ hr. } 60 \text{ min.}} \end{array}$$

$$\begin{array}{r} 4 \\ 75 \text{ min.})\overline{300 \text{ min.}} \\ \underline{300 \text{ min.}} \end{array}$$

Find the answers. Think what each answer means.

1. How many pieces of wood 2 ft. 6 in. long can be cut from a board 15 ft. long?

$$\begin{array}{l} 2 \text{ ft. } 6 \text{ in.} = \underline{?} \text{ in.} \\ 15 \text{ ft.} = \underline{?} \text{ in.} \end{array}$$

$$30 \text{ in.})\overline{180 \text{ in.}}^{\,?}$$

2. 30 ft. ÷ 2 ft. 6 in.
3. 11 pk. ÷ 1 pk. 3 qt.
4. 21 yd. ÷ 2 yd. 1 ft.
5. 14 hr. ÷ 2 hr. 20 min.
6. 28 qt. ÷ 3 qt. 1 pt.
7. 18 lb. ÷ 2 lb. 4 oz.

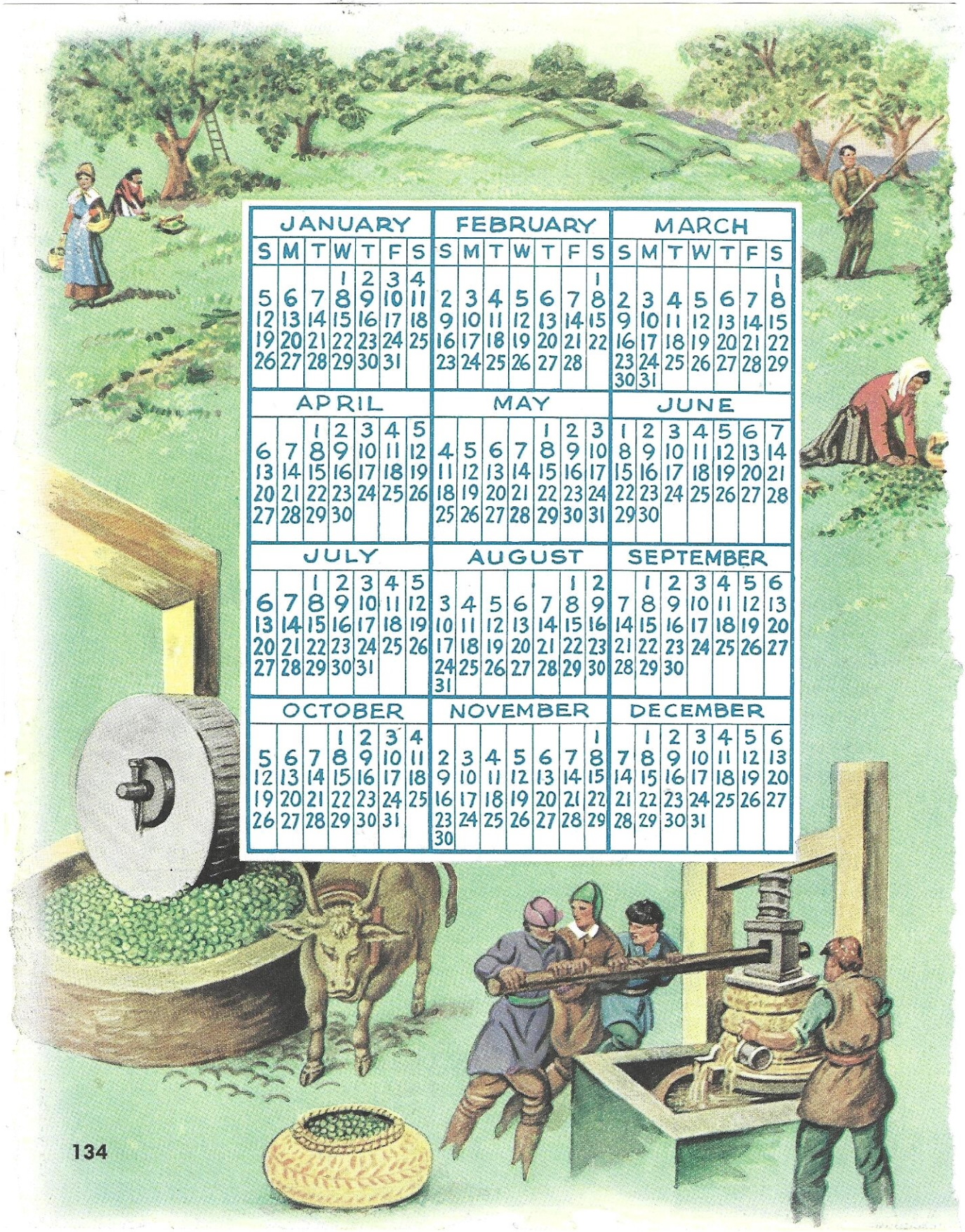
Look back at your answer for example 7. Then study the check below.

Check for example 7:

$$\begin{array}{r} 36 \text{ oz.} \\ \times 8 \\ \hline 288 \text{ oz.} \end{array} \qquad \begin{array}{r} 18 \\ 16 \text{ oz.})\overline{288 \text{ oz.}} \\ \underline{16} \\ 128 \\ \underline{128} \end{array}$$

18 groups of 16 oz. = 18 lb.

8. Check your answers for examples 1–6.



JANUARY

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

FEBRUARY

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

MARCH

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

APRIL

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

MAY

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

JUNE

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

JULY

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

AUGUST

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

SEPTEMBER

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

OCTOBER

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

NOVEMBER

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

DECEMBER

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

Table of Time

7 days (da.) = 1 week (wk.)	365 days = 1 year
52 weeks = 1 year (yr.)	366 days = 1 leap year
12 months (mo.) = 1 year	100 years = 1 century

Larger Measures of Time

DAY, WEEK, MONTH, YEAR, CENTURY

1. The year 1960 was a leap year. The numbers for leap years can be divided exactly by 4.

   1960 ÷ 4 = _?_. Which of the following are leap years? 1492, 1776, 1812, 1886, 1964

2. In leap year an extra day is added to February. How many days does February have in a leap year?

3. In an olive orchard, olives were ripe enough to pick on October 1. What day of the week is this on the calendar shown here?

4. The trees in this orchard bore olives until December 31. What day of the week is December 31 on the calendar shown here?

5. The orchard had ripe olives from October 1 to December 31. How many months is this? How many days is it?

6. In an orchard of younger trees the picking season started on October 20 and lasted 53 days. On what date was the season over? On what day of the week on the calendar shown here did this date fall?

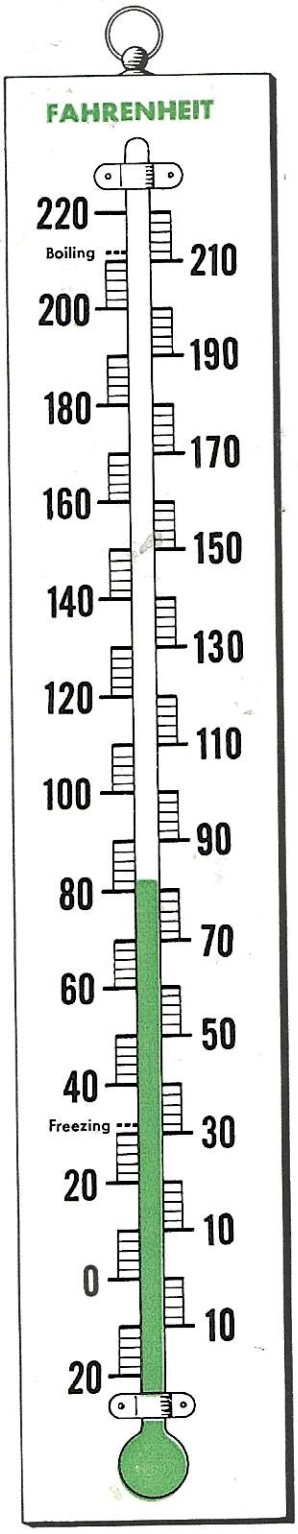
7. Olive trees often bear olives for a century. How many years is a century? Explain the old saying, "If you would plant wealth for your great-grandchildren, plant the olive."

8. In ancient orchards some giant olive trees are 7 centuries old. How many years is 7 centuries?

9. Why do we say that we are living in the twentieth century?

10. An olive tree that is now 7 centuries old was planted in the _?_ century.

11. An olive tree does not have a full crop of olives until it is 30 years old. If an olive tree is planted this year, in what year can it be expected to have a full crop?



Thermometer

A *thermometer* is used to measure *temperature*. Temperature is measured in degrees (°).

Look at the drawing of the thermometer at the left.

1. The space between each two numbers on the thermometer stands for _?_ degrees. This space is divided into _?_ equal parts. Each part stands for _?_ degrees. One degree would be shown by _?_ of one small space.

The temperature at which pure water freezes is called the *freezing point*. Read the freezing point on the thermometer. Thirty-two degrees above zero may be written 32° or +32°. Any temperature above zero may be written with a *plus sign*. When the liquid in the tube is lower than 0°, the temperature is "below zero." Two degrees below zero may be written −2°. Any temperature below zero may be written with a *minus sign*. When a temperature is written without any sign or without saying "below zero," we assume that it means above zero.

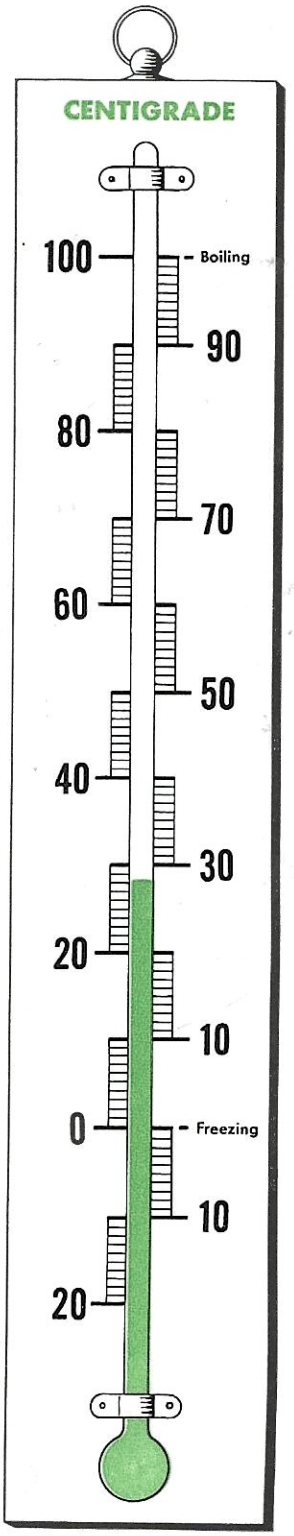
At normal pressure, pure water boils at 212°. This is called the *boiling point*. Find 212° on the drawing of the thermometer.

2. How many degrees are there between the freezing point and the boiling point?
3. An olive tree would be injured if the temperature were −6°. How many degrees below the freezing point is this?
4. One day last winter, it was −5° at 6 A.M. It was +10° at 11 A.M. How many degrees had the temperature risen in five hours? This was an average of how many degrees per hour?
5. Read the temperature shown on the thermometer in the drawing. If the temperature rose 1°, where should the top of the liquid be?

More about Measures

ENRICHMENT

Most household thermometers use the *Fahrenheit* scale as shown on the thermometer on page 136. In scientific work, a *centigrade* thermometer is often used.



1. *Centi* means *100; grade* means *steps.* On the centigrade scale, water freezes at 0° and boils at 100°. Do you see 100 steps?

2. Is 0° centigrade the same temperature as 32° Fahrenheit? Is 100° centigrade the same temperature as 212° Fahrenheit?

3. How many degrees are there between the freezing point and the boiling point on the centigrade scale? On the Fahrenheit scale? Then one degree centigrade would be equivalent to how many degrees Fahrenheit?

4. The normal temperature of the human body is 37° centigrade. Use your answer to the last part of question 3 to change this to the equivalent temperature on the Fahrenheit scale. After you multiply, you must add 32° because that is the Fahrenheit reading that corresponds to 0° centigrade. Is your answer sensible?

5. The Indians measure months by the moons. Use a calendar to find out how many days it is from one full moon to the next full moon. Is the time always the same? How does this compare with our system of measuring months?

6. If you had a very carefully constructed ruler, could you draw a line exactly 1 inch long? Might it be $\frac{1}{1000}$ inch too long or too short? Can you measure exactly 1 quart of milk? Can you measure exactly 1 bushel of apples? Can you measure these things close enough for your purpose? Explain the statement *all measurement is approximate.*

Practice to Remember

MAINTENANCE TEST ON MEANINGS AND SKILLS (CHAPTERS 1–3)

Add.

1.

672	796	802	2679	8769
207	872	706	3248	7458
56	30	893	5006	6872
439	42	347	6709	7541
116	569	656	5368	9892

Subtract.

2.

605	823	765	1682	3000
413	746	30	897	1628

Multiply.

3.

76	408	567	756	996
42	723	820	879	843

4.

632	721	301	981	156
123	243	157	879	878

Divide.

5. $21\overline{)1581}$ $88\overline{)3676}$ $79\overline{)52642}$ $31\overline{)859}$

6. $92\overline{)1319}$ $84\overline{)1797}$ $78\overline{)2513}$ $61\overline{)1997}$

Add. Change answers to lowest terms.

7.

$2\frac{1}{3}$	$4\frac{2}{5}$	$4\frac{3}{4}$	$6\frac{1}{2}$	$7\frac{2}{7}$	$9\frac{5}{6}$
$4\frac{5}{9}$	$7\frac{3}{10}$	$9\frac{7}{8}$	$8\frac{3}{4}$	$4\frac{5}{14}$	$5\frac{5}{12}$

Subtract. Change answers to lowest terms.

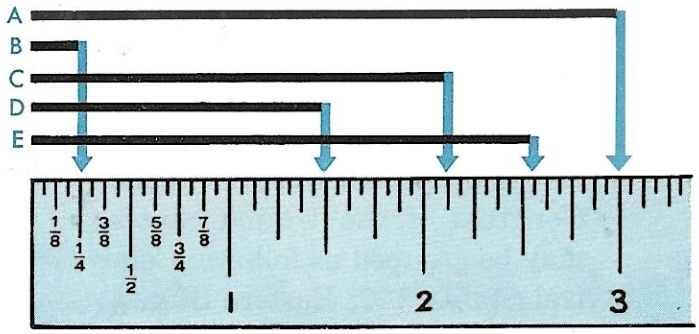
8.

$7\frac{3}{4}$	$9\frac{5}{8}$	$4\frac{5}{6}$	6	$4\frac{1}{8}$	$7\frac{1}{6}$
$3\frac{1}{4}$	$2\frac{3}{4}$	$2\frac{1}{3}$	$3\frac{1}{2}$	$\frac{3}{4}$	$4\frac{5}{6}$

Practice to Remember

MAINTENANCE TEST ON PROBLEM SOLVING (CHAPTERS 1–3)

1. Mary wanted to buy a bicycle so she could ride to school. She counted her savings and found she had $56.47. How much more did she need in order to buy a bicycle priced at $65.00?
2. What was the Akin School Cafeteria's milk bill for February if, for the first week, it was $63.75; the second week, $59.35; the third week, $75.50; and the fourth week, $72.25?
3. In a certain contest Henry won a prize which gave him $75 worth of clothing at a local store. He spent $13.75 for shirts; $26.80 for a suit; $6.00 for socks; $8.75 for ties; and $5.30 for underwear. How much of his prize money did he spend?
4. A large four-engined navy flying boat made a record nonstop flight of 4748 miles in approximately 24 hours. What was its average speed for the flight?
5. A tropical hurricane was discovered in the South Atlantic traveling at approximately 18 miles an hour. How far would this hurricane travel in a day? (_?_ hours = 1 day)
6. The number of members in the House of Representatives in the Congress of the United States may be grouped as follows: Central States, 122; Eastern States, 121; Southern States, 126; and Western States, 66. What is the total number of representatives?
7. Mr. Harris had 214 acres planted in corn. He expected an average of 37 bushels to the acre. How many bushels of corn did he expect to get?
8. Mr. and Mrs. James drove 1568 miles in 4 days. What average distance did they drive per day?
9. Mr. Harmon used 26 gallons of gasoline in driving his car from New York City to Charlestown, West Virginia. The total distance traveled was 598 miles. What was the average number of miles he traveled on each gallon?
10. During a certain year the United States had 227,335 miles of railroad. In the entire world at that time there were 788,672 miles of railroad. How many miles of railroad were there in countries other than the United States?



Using a Ruler

It is important to know how to use a ruler. We use a ruler to find how long something is or to draw line segments.

Look at your ruler or the drawing of the ruler at the left.

1. Point to the marks that show whole inches; half inches; quarter inches; eighth inches. What do the very shortest marks show?
2. How long is bar A? Bar B? Bar C? Bar D?
3. Suppose that you want to measure bar E to the nearest quarter inch. The end of bar E comes between $2\frac{1}{2}$ inches and $2\frac{3}{4}$ inches. Is a half-inch mark also a quarter-inch mark? Is the end of bar E nearer to $2\frac{1}{2}$ inches than to $2\frac{3}{4}$ inches? Is it less than halfway? Then bar E, to the nearest quarter inch, measures _?_ inches.
4. With your ruler, measure the lengths of the following line segments to the nearest quarter inch.

5. Draw a line segment whose length is 2 inches; $1\frac{1}{2}$ inches; $3\frac{1}{4}$ inches; $4\frac{3}{4}$ inches; $2\frac{1}{8}$ inches; $2\frac{5}{8}$ inches; $3\frac{7}{8}$ inches. Next to each line segment write its length.
6. Do you think that one of your hairs is less than $\frac{1}{16}$ inch thick? Put one against your ruler and see. Can you really measure the thickness of a hair with your ruler? Why not?
7. How many inches long is your foot? Is it a foot long? Make a guess. Then measure it with your ruler.

Shapes and Sizes

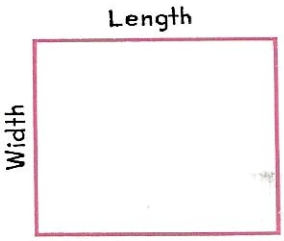
RECTANGLES, TRIANGLES, DIMENSIONS

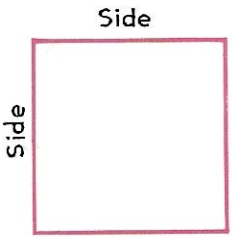
A *rectangle* has four straight sides and four square corners. Its opposite sides are equal.

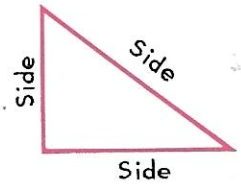
A *square* is a rectangle with all sides equal.

The length and width of a rectangle are called its *dimensions*. We may say that a rectangle that is 3 inches long and 2 inches wide is 3 in. by 2 in. or $3'' \times 2''$. The sign ($''$) means *inches*. The sign $\times$, used with dimensions, means *by*.

A *triangle* has three straight sides. The three sides may or may not be equal.





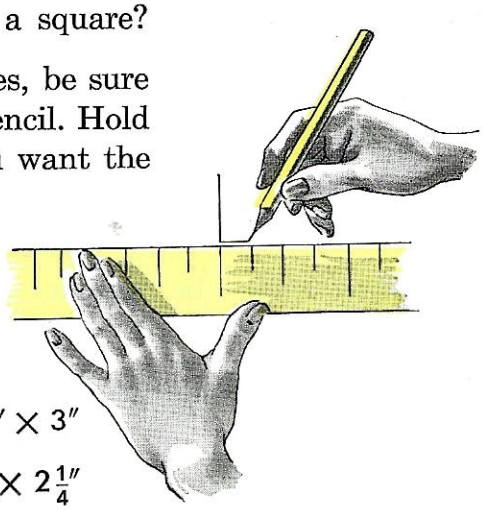


1. Measure the sides of the three figures shown at the right. Write the dimensions of the rectangle and the square using the signs for *inches* and for *by*.

2. Write the lengths of the three sides of the triangle in a column. Find their sum.

3. Measure the dimensions of your arithmetic book cover. Write them. Is the shape a rectangle? Is it a square?

When you use a ruler to draw connecting lines, be sure to allow for the thickness of the point of your pencil. Hold the ruler just a little below the place where you want the line to be.



You can use your ruler to make a square corner by placing one of the longer lines on the ruler so that it makes a straight line with the line on your paper, as shown at the right.

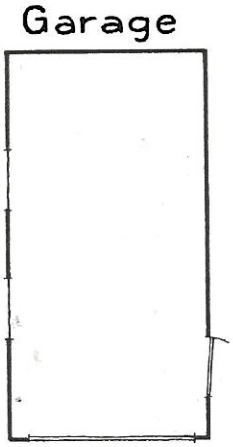
4. Draw rectangles with these dimensions: $4'' \times 3''$

   $5'' \times 5''$ $\quad 5\frac{1}{4}'' \times 3''$ $\quad 4\frac{3}{4}'' \times 3\frac{1}{4}$ $\quad 2\frac{1}{4}'' \times 2\frac{1}{4}''$

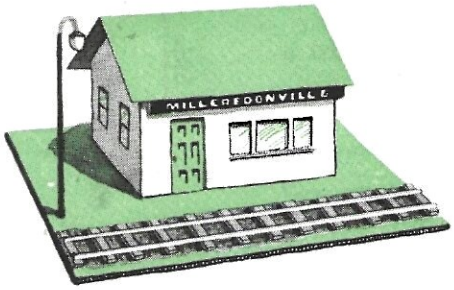
5. Which of the figures you just drew are squares?

6. Make a list of things that are shaped as a rectangle; a square; a triangle.

Drawing and Building to Scale



Scale 1" = 10'

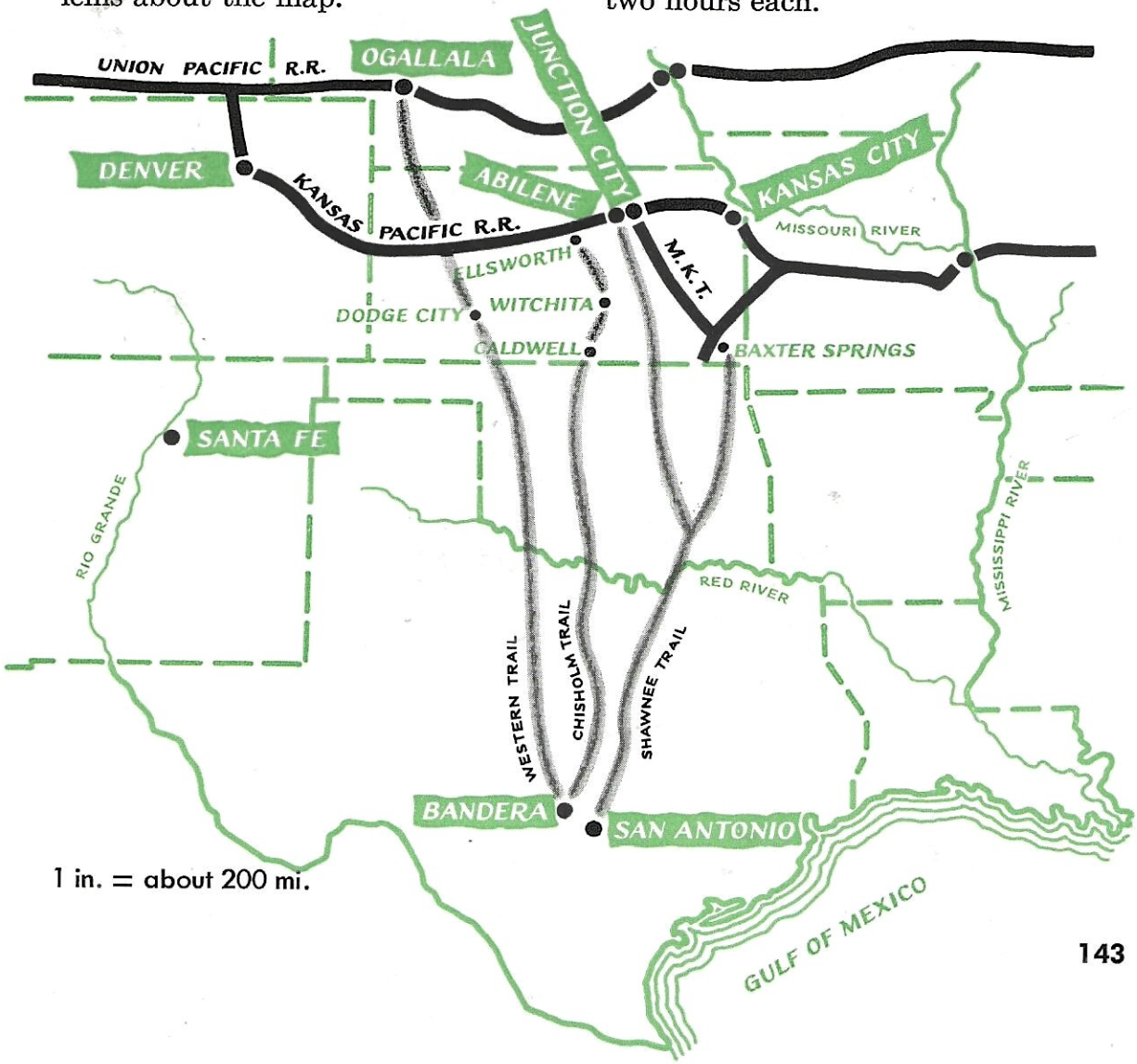


1. Henry made a *scale drawing* of his father's garage. He let 1 inch represent 10 feet. Explain the way he wrote the scale below his drawing. The sign (') means *feet*.
2. Measure the sides of the drawing. How many inches wide is the drawing? Then how wide is the garage?
3. How many inches long is the drawing? Then how long is the garage?
4. Write the dimensions of the garage using the sign for *feet* and for *by*.
5. Make a scale drawing of the same garage with the scale 1″ = 5′. What are the dimensions of your drawing?
6. Jerry wants to make a cardboard scale model of the local railroad station for an exhibit at school. The station is 20 ft. long and 15 ft. wide. Jerry is going to use the scale 1 in. = 2 ft. Then 10 in. = _?_ ft. Jerry's model will be _?_ in. long. If 1 in. = 2 ft., what part of an inch will represent 1 ft.? How will you find how many inches will represent 15 ft.? In 15 ft. there are $7\frac{1}{2}$ groups of 2 ft. Jerry's model will be _?_ wide.
7. The ticket office in the station is a little room 6′ × 8′. What will be the dimensions of the ticket office in Jerry's model?
8. Ann wants to make a scale drawing of her classroom. The room measures 24′ × 36′. The paper Ann is using is $8\frac{1}{2}$″ × 11″. If she uses the scale 1″ = 3′, will her drawing fit on her paper? Could she use 1″ = 4′?
9. Make a scale drawing of your classroom. Use the scale that will make your drawing fit best on your paper.

Reading a Map

This is a page to have fun with. Sixth graders know how to read maps. Notice the scale to which this map has been drawn. Decide what $\frac{1}{2}$ inch, $\frac{1}{4}$ inch, and $\frac{1}{8}$ inch will stand for. Think about whether it is possible to find exact distances along a line that is not straight. Then make up and solve some problems about the map.

Here is some extra information for you: In the Old West, cattle were driven great distances from the ranches to the railroad centers. There were several famous trails. The cattle traveled 10 to 15 miles per day. The cowboys guarded the cattle day and night. The night guards worked in four shifts of two hours each.

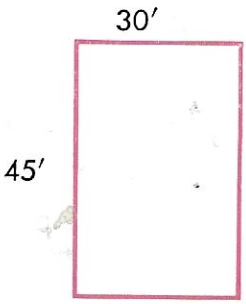


1 in. = about 200 mi.

Finding Perimeters

The sixth graders were working problems in finding perimeters. They knew that the *perimeter* of a figure is the sum of the lengths of its line segments.

Charles said, "You can find the perimeter of any figure by adding the measurements of all its sides."



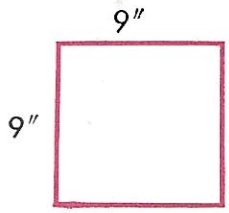
To find the perimeter of the rectangle at the left, Charles added. $45' + 30' + 45' + 30' =$?

Ann thought it would be shorter to do this: $45' + 30' = 75'$; $2 \times 75' =$?

Jim did it this way: $2 \times 45' = 90'$; $2 \times 30' = 60'$; $90' + 60' =$?

Are all three ways correct? Choose the way you like best. Could you combine the numbers in these ways if the dimensions were not all in the same unit of measure?

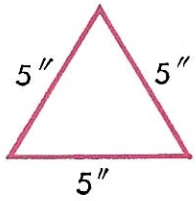
1. Find the perimeter of rectangles with the following dimensions: $9' \times 18'$ 4 yd. $\times$ 7 yd. $23'' \times 35''$ 15 rd. $\times$ 20 rd.



2. Find the perimeter of your desk. Measure each dimension to the nearest whole inch.

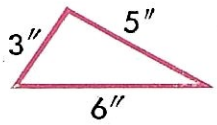
3. A box top is 9 inches square. What is its perimeter? Find the answer by adding. Find it by multiplying. Make a rule for finding the perimeter of a square.

4. Find the perimeter of the following squares: $11' \times 11'$ $30'' \times 30''$ 15 rd. $\times$ 15 rd. 5 yd. $\times$ 5 yd.



5. The perimeter of a square is 12 yards. Find the length of one side.

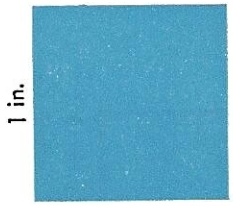
6. Find the perimeter of a triangle that measures 5 inches on each side. Find it by adding. Find it by multiplying.



7. Find the perimeter of a triangle that has one side 3″ long, another side 5″ long, and another side 6″ long. Find it by adding. Can you find it by multiplying? Why?

Area

SQUARE INCHES



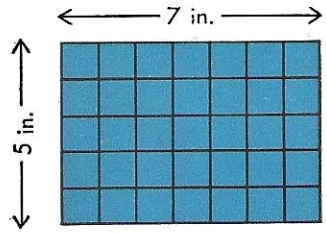
John has a card 7 inches long and 5 inches wide. Bill has a card 8 inches long and 4 inches wide. Which card is larger?

To find which card is larger, the boys need to find the area of each card and then compare the areas.

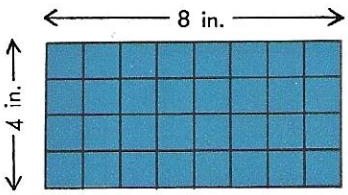
Area is amount of surface. To find the area of each card in square inches, John and Bill each cut a square inch of paper.

The square at the top of the page measures 1 inch on each side. It will cover an area of 1 square inch.

Each boy placed a square inch of paper on his card and drew thin lines around it. He covered his card in this way so that, when finished, the two cards looked like the drawings at the right.



John's Card
Area: 35 square inches



Bill's Card
Area: 32 square inches

Scale of both drawings: $\frac{3}{16}'' = 1''$

John and Bill counted the number of square inches marked on their cards. John's card has 35 square inches. Bill's card has 32 square inches. John's card is 3 square inches larger than Bill's.

Could the boys find the areas by multiplying?

John's card has 5 rows of 7 one-inch squares.
5×7 square inches = 35 square inches.

Bill's card has 4 rows of 8 one-inch squares.
4×8 square inches = 32 square inches.

1. Make a rule for finding the area of a rectangle when both the length and width are given in inches.

Use your rule to find the areas in square inches of rectangles with the dimensions given below. Draw pictures to help you, if necessary.

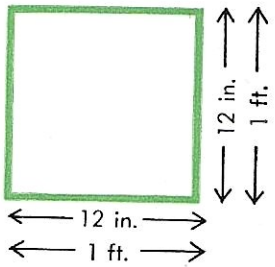
2. 4 in. by 9 in.
6 in. $\times$ 8 in.
$1'' \times 10''$

3. 20 in. by 3 in.
12 in. $\times$ 2 in.
$9'' \times 7''$

4. 30 in. by 4 in.
5 in. $\times$ 20 in.
$10'' \times 10''$

Square Measure

SQ. IN., SQ. FT., SQ. YD.

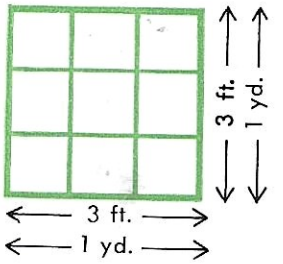


The drawing at the left represents a square measuring 1 foot on each side, enclosing an area of 1 square foot.

1. Cut a square 1 foot on a side out of paper or cardboard. Can you find how many square inches are in a square foot? Draw vertical and horizontal lines 1 inch apart on your square if you need them.

Think: There are _?_ 1-inch squares along the top of the square foot. Then _?_ rows of 1-inch squares will fit across the side. To find the area, I _?_.

12×12 square inches = _?_ square inches

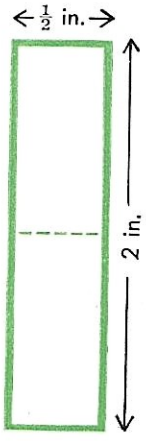


The drawing at the left represents a square measuring 1 yard on each side. It will cover an area of 1 square yard.

2. Draw a square 1 yard on a side on the blackboard or on the floor. Using the square you made for example 1, mark off this square yard into square feet as shown in the drawing.

To find how many square feet are in a square yard, think how many squares 1 foot on each side are needed to cover 1 square yard.

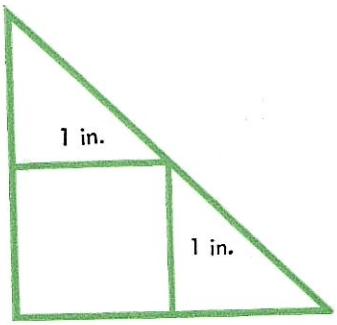
1 square yard = 3×3 square feet = _?_ square feet



144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)

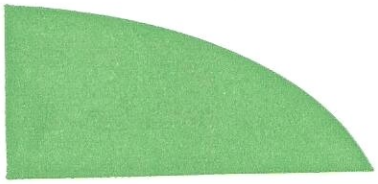
To have its area measured in square units such as square inches, square feet, or square yards, a figure need not be square in shape. It need not even have straight sides. Any area that may be cut up and fitted together a different way to cover a square 1 inch on a side has an area of 1 square inch. For example, a rectangle that is 2 inches long and $\frac{1}{2}$ inch wide has an area of 1 square inch.

3. Mark off 2 inches on each side of the corner of a piece of paper. Cut along the line connecting these two marks. Now you have a triangle with a base of 2 inches and a height of 2 inches. Draw a 1-inch square in your triangle as shown in the drawing. Work carefully so that the lines you draw complete a true square.



Cut your triangle apart along the lines you have drawn. Fit the square and the two other pieces together to show that your triangle has an area of 2 square inches.

4. Phil's house is on a lot shaped like the drawing at the right. The little map that came with the lot said, "Area—13,444 sq. ft." Explain how this can be so.



It is easy to find the area of a rectangle. Think of the rectangle as being divided into small squares and you will know how to find its area.

5. Suppose that each square in the drawing at the right is 1 inch on a side. The area of each square is 1 _?_ _?_. In the rectangle there are _?_ rows with 2 square inches in each row. The area of the rectangle is 4×2 sq. in. = _?_ sq. in.

6. Suppose that each square in the drawing is 1 foot on a side. Explain how to find its area.

7. Make up a problem about finding the area of a rectangle whose dimensions are given in yards. Draw a picture of your rectangle. Find its area.

8. Do you think the rule given below would be true if the dimensions of the rectangle were not both whole numbers?

To find the area of a rectangle, multiply the number of units in its length by the number of units in its width.

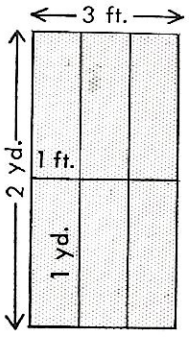
Finding Areas

1. Complete: To find the area of a rectangle, multiply _?_.

Find the areas of the following rectangles. Remember to write the answers in units of square measure. If the dimensions are given in feet, give the answer in square feet. If the dimensions are given in inches, give the answer in _?_ _?_. If the dimensions are given in yards, give the answer in _?_ _?_.

2.	5 ft. by 8 ft.	13′ × 7′	3 yd. by 15 yd.
3.	10 in. by 12 in.	15″ × 18″	22 yd. by 19 yd.
4.	22 in. by 19 in.	32″ × 34″	42 yd. by 35 yd.
5.	36 ft. by 74 ft.	47′ × 63′	1 yd. by 15 yd.

6. Which has a larger area: a room 15 feet by 12 feet, or a room 14 feet by 14 feet? A room 24 feet by 18 feet, or a room 22 feet by 20 feet?



If the area of a rectangle is to be measured in square units, the dimensions must be in the same unit. The drawing at the left shows why this is so. If you have feet in one direction and yards in the other, are the units of area square?

When the dimensions of a rectangle are given in different units, you must change one of them so that the units are alike.

7. Find the number of square feet in the area of a rectangle 36 inches by 2 feet.
First change 36 inches to _?_ feet. The area is _?_ sq. ft.

8. Find the area of the same rectangle in square inches.

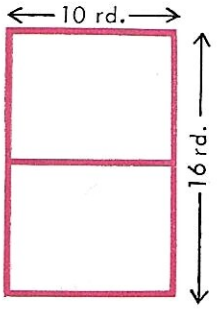
9. Find the area in square feet of the following rectangles:
24 in. by 4 ft. 36″ by 6′ 72″ × 16′.

10. Find the area in square yards of the following rectangles:
9 ft. by 12 yd. 6 ft. × 6 yd. 108 in. × 14 yd.

Rods, Acres, Square Miles, Sections

Large areas are measured in square rods, acres, square miles, and sections.

160 square rods (sq. rd.) = 1 acre (a.)
640 acres = 1 square mile (sq. mi.) = 1 section



1. Mr. Kulp owns an acre of land. It is 16 rods by 10 rods. How many square rods does it contain?
2. If Mr. Kulp divides this acre into building lots, each 10 rods by 8 rods, how many building lots will he have?
3. Mr. Miller has a meadow 20 rods by 16 rods. How many square rods are in the meadow? How many acres is this?
4. Mr. Treat has a farm which measures 64 rods by 50 rods. The farm has _?_ square rods, or _?_ acres.
5. Mr. Kleir owns a $\frac{1}{4}$-acre lot. How many square rods are in his lot?
6. You have made actual size drawings of a square inch, a square foot, and a square yard. Can you draw a square rod on your playground? Can you draw a square mile in actual size? Can you picture a square mile in your mind? If you made a square of any size and labeled each dimension *1 mile*, would your square represent 1 square mile?
7. How would you find the area of a rectangle whose dimensions were given in miles? In what units would you give your answer?
8. A ranch is 3 miles long and 2 miles wide. How many square miles is the ranch? How many sections? How many acres?
9. A farm $\frac{1}{8}$ square mile in area has _?_ acres.
10. In a reference book, look up the area of your state. In what units is the area given? Explain how the area of your state can be measured in square miles.

Changing Square Measures

Units of square measure are changed into other units by the same methods used in changing any similar units of measure.

For example, if you want to find how many sq. yd. there are in 18 sq. ft., think as follows:

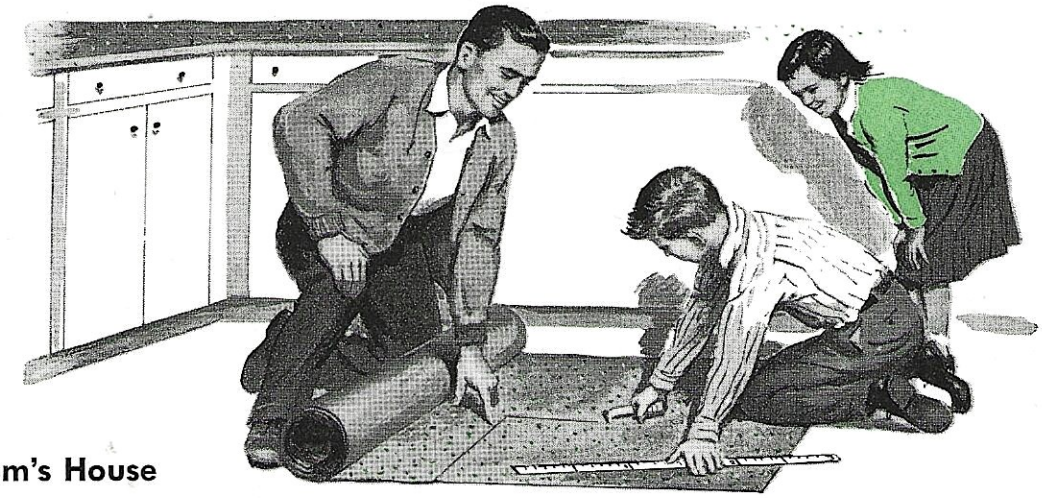
"Is a square foot smaller than a square yard? Does it take more than 1 square foot to make 1 square yard? How many square feet are there in a square yard?" (If you do not remember, draw a picture like one of those on page 146.) "I must do something with this 9. Shall I multiply or divide?

"If I multiply by a number larger than 1, is the answer larger than the number I multiplied? If I divide by a number larger than 1, is the answer smaller than the number I divided?

"In this case, there will be fewer square yards than square feet. I divide to find the answer. $18 \div 9 = 2$. 18 sq. ft. = 2 sq. yd."

Use the same method of thinking to do the following examples and problems.

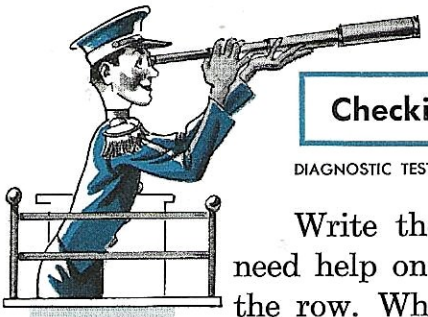
1. 63 sq. ft. = _?_ sq. yd. — 6 sq. ft. = _?_ sq. yd.
2. 10 sq. yd. = _?_ sq. ft. — $\frac{1}{3}$ sq. yd. = _?_ sq. ft.
3. 432 sq. in. = _?_ sq. ft. — 864 sq. in. = _?_ sq. ft.
4. 2 sq. ft. = _?_ sq. in. — $\frac{1}{2}$ sq. ft. = _?_ sq. in.
5. 1280 acres = _?_ sq. mi. — 80 acres = _?_ sq. mi.
6. 3 sq. mi. = _?_ acres — $\frac{1}{4}$ section = _?_ acres
7. 6 sq. ft. = _?_ sq. in. — 11 sq. yd. = _?_ sq. ft.
8. Mrs. Jay wants a rug to cover a floor 9 ft. by 12 ft. The kind of rug she wants is sold by the square yard. How many square yards will she need?
9. Can you think of two ways to do problem 8? Which way is easier? Why? Would this way always be easier if the numbers in the problem were changed?



At Tom's House

PERIMETER AND AREA

1. Perimeter is the length of a line. You use _?_ measure in finding perimeters.
2. Area covers all of a surface. You use _?_ measure in finding areas.
3. The lot on which Tom's father built his house is 125 feet long and 75 feet wide. What is the perimeter of the lot? What is the area?
4. The lawn in front of Tom's house is 75 feet by 51 feet. The perimeter of the lawn is _?_ feet. The lawn contains _?_ square feet, or _?_ square yards.
5. The house is 35 feet by 24 feet. It occupies an area of _?_ square feet. The perimeter of the house is _?_ feet.
6. The dimensions of the kitchen floor are 12 feet by 9 feet. What is its area in square feet? In square yards?
7. Tom's father purchased 12 square yards of linoleum at $6.00 per square yard. How much did the linoleum cost?
8. A large pane of glass in the front window is 40 inches by 36 inches. How many square inches are there in its surface? Change the square inches into square feet. Is this surface more or less than 1 square yard?
9. The garden is 27 feet by 15 feet. What are the dimensions of the garden in yards? The area of the garden is _?_ square yards.
10. What is the perimeter of a garden 27 feet by 15 feet? At 80 cents a foot, how much will it cost to put a wire fence around the garden?



Checking Up on Measures

DIAGNOSTIC TEST ON PP. 121–149

Write the answers to the following exercises. If you need help on any exercise, turn to the page shown beside the row. When you change measurements in one unit to another unit, think whether your answer should be larger or smaller than the number you had at first. Then you can decide whether to multiply or divide.

Copy and fill in the missing numbers.

121	1. 1 pt. = _?_ c.	8 qt. = _?_ gal.
121	2. 2 qt. = _?_ pt.	1 c. = _?_ fl. oz.
122	3. 24 in. = _?_ ft.	72 in. = _?_ yd.
122	4. 6 ft. = _?_ yd.	1 mi. = _?_ ft.
126	5. 1 c. = _?_ tbs.	3 tsp. = _?_ tbs.
126	6. 3 tbs. = _?_ tsp.	8 tbs. = _?_ c.
127	7. 4 pk. = _?_ bu.	2 pk. = _?_ bu.
127	8. 2 pk. = _?_ qt.	16 pt. = _?_ qt.
128	9. $\frac{1}{2}$ lb. = _?_ oz.	12 oz. = _?_ lb.
128	10. 1 T. = _?_ lb.	4000 lb. = _?_ T.
129	11. 24 hr. = _?_ da.	2 hr. = _?_ min.
129	12. $\frac{1}{2}$ min. = _?_ sec.	30 min. = _?_ hr.
135	13. 1 yr. = _?_ wk.	6 mo. = _?_ yr.
135	14. 30 mo. = _?_ yr.	56 da. = _?_ wk.
146	15. 1 sq. ft. = _?_ sq. in.	$\frac{1}{4}$ sq. ft. = _?_ sq. in.
146	16. 1 sq. yd. = _?_ sq. ft.	27 sq. ft. = _?_ sq. yd.
149	17. 1 sq. mi. = _?_ a.	80 a. = _?_ sq. mi.
149	18. 1 a. = _?_ sq. rd.	3 sq. mi. = _?_ sections

Copy and add. Be sure your answers are in simplest form.

19. 3 ft. 6 in. / 2 ft. 3 in. — 2 gal. 3 qt. / 1 gal. 1 qt. — 1 yd. 2 ft. / 3 yd. 2 ft. 124

Copy and subtract. Check by going over your work.

20. 3 qt. 1 pt. / 1 qt. 1 pt. — 3 ft. 5 in. / 1 ft. 7 in. — 3 lb. 2 oz. / 2 lb. 8 oz. 125

Copy and multiply.

21. 1 hr. 15 min. ×2 — 2 da. 10 hr. ×3 — 4 min. 30 sec. ×5 131

Copy and divide.

22. $2\overline{)5 \text{ wk. } 3 \text{ da.}}$ $4\overline{)5 \text{ ft. } 8 \text{ in.}}$ $3\overline{)10 \text{ sq. yd. } 2 \text{ sq. ft.}}$ 132

Find the perimeters of the following figures:

23. Rectangle, 10′ × 8′ — Square, 7″ on a side — Triangle, 16″ on a side 144

Find the areas of rectangles with the following dimensions:

24. 4 ft. by 8 ft. — 9 yd. by 18 yd. — 20″ × 22″ 147

25. How many square feet of area has a rectangle that measures 8 feet by 2 yards? 148

Just for Fun

1. Hold out your arms at the side as straight as you can hold them. Have someone measure the distance between the tips of your two longest fingers. Is this distance about equal to your height?
2. Bend your elbow. Measure the distance between the tip of your elbow and your wrist bone. How does this distance compare with the length of your foot?
3. Would it be possible to draw a life-size picture of your face on a piece of paper the size of a page of your arithmetic book?

What Have You Learned about Measures?

CHAPTER TEST ON MEANINGS AND SKILLS

1. Write the unit of measure you would use for measuring:
   (a) Distance between cities
   (b) Area of a farm
   (c) Gingham for a dress
   (d) Temperature of water
   (e) Weight of a baby
   (f) Vanilla flavoring
   (g) Weight of a large truck
   (h) Container of milk

2. Write three units of each of the following kinds of measures:
   (a) Weight
   (b) Length
   (c) Area
   (d) Time
   (e) Liquid
   (f) Dry

For each blank in exercises 3–5, use one of the following words: *rectangle, perimeter, square, triangle, area, dimensions.*

3. A figure with four straight sides and four square corners is called a _?_. If the four sides are equal, it is called a _?_.
4. The total length of the sides of a triangle is called the _?_.
5. The width and length of a rectangle are called its _?_.
6. One cup = _?_ fluid ounces, so 5 c. = _?_ fl. oz. and 24 fl. oz. = _?_ c.
7. One yard = _?_ inches, so 4 yd. = _?_ in. and 72 in. = _?_ yd.
8. One peck = _?_ quarts, so 6 pk. = _?_ qt. and 40 qt. = _?_ pk.
9. One pound = _?_ ounces, so 9 lb. = _?_ oz. and 80 oz. = _?_ lb.
10. One hour = _?_ minutes, so 3 hr. = _?_ min. and 240 min. = _?_ hr.
11. How many degrees has the temperature risen when the thermometer reading moves from 2 degrees below zero to 8 degrees above zero?
12. One square foot = _?_ square inches, so 5 sq. ft. = _?_ sq. in.
13. One square yard = _?_ square feet, so 36 sq. ft. = _?_ sq. yd.

14. Using the scale $1'' = 10'$, draw lines to represent the following lengths:

    $10'$ $20'$ $30'$ $15'$ $25'$ $12\frac{1}{2}'$

15. Find the perimeter of a rectangle $16'$ by $24'$.
16. Find the area of a rectangle 15 yd. by 25 yd.
17. Add 3 gal. 3 qt. and 2 gal. 2 qt.
18. Subtract 2 lb. 9 oz. from 3 lb. 3 oz.
19. Multiply 1 hr. 15 min. by 4.
20. Divide 6 ft. 9 in. by 3.

What Have You Learned about Measures?

CHAPTER TEST ON PROBLEM SOLVING

1. The Weber family has moved into their new home. The grounds cover $\frac{1}{4}$ of an acre. How many square rods is this?
2. Mr. Weber is making a play yard 15 feet by 10 feet for two-year-old Teddy. How many feet of fencing will he need to enclose it? How many square feet of play space will it have?
3. Arthur is putting up his tepee. It has a base 5 feet by 5 feet. How many square feet are there inside the base of the tepee?
4. The Webers are planning a garden. They will plant 3 rows of string beans. Each row will be 12 feet 6 inches long. How long will all 3 rows together be?
5. One package of radish seeds will plant a row 25 feet long. Mr. Weber wants to plant 2 rows each 12 feet 6 inches long. Will one package of seeds be enough?
6. Arthur started to work in the garden at 9:15 A.M. He stopped at 11:45 A.M. How long did he work?
7. Jane is making pudding. The recipe calls for 2 cups of milk. How many pints of milk will she use? What part of a quart is that?

CHAPTER 5

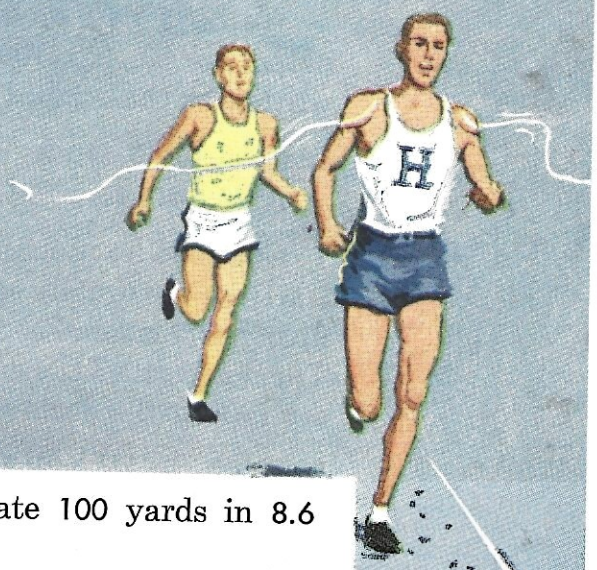
Decimals Are Used in Sports

If you are interested in sports records, you must know how to use decimals.

Can you read the numbers in the sentences below? Do you know what they mean?

1. A boy in New Jersey won a National Amateur Bicycle Championship Race by riding 1 mile in 3 minutes 8.1 seconds.
2. A champion swimmer can swim 100 yards in 57.4 seconds.

Adding and Subtracting Decimals



3. A very good speed skater can skate 100 yards in 8.6 seconds.
4. The High School National Interscholastic Track record for the 100-yard dash is 9.4 seconds.
5. A skier in Austria has jumped 350.96 feet.
6. The winning bobsled team ran the course in 5 minutes 7.85 seconds.

	8.1	57.4	8.6	9.4	350.96	7.85
means	$8\frac{1}{10}$	$57\frac{4}{10}$	$8\frac{6}{10}$	$9\frac{4}{10}$	$350\frac{96}{100}$	$7\frac{85}{100}$

Decimals

VOCABULARY; MEANING; READING AND WRITING TENTHS AND HUNDREDTHS

We have two ways of writing fractions whose denominators are 10, 100, 1000, and the like. Three tenths may be written as the *common fraction* $\frac{3}{10}$ or as the *decimal fraction* .3.

1. $\frac{3}{10}$ means 3 tenths, so .3 means _?_.
   $\frac{7}{10}$ means _?_, so .7 means _?_.

Common fractions are usually called *fractions*, and decimal fractions are usually called *decimals*. A number like 4.9, made up of a whole number and a decimal, is called a *mixed decimal*. The dot is called the *decimal point*. In reading mixed decimals, say *and* for the decimal point.

2. Copy the following table leaving blanks where there are question marks. Then fill in the blanks so that each column will show two ways of writing the same number. Read each number.

fraction	$\frac{2}{10}$	$\frac{6}{10}$	$\frac{9}{10}$	?	?	$1\frac{1}{10}$	$1\frac{2}{10}$	$3\frac{7}{10}$	?	?
decimal	.2	?	?	.7	.5	1.1	?	?	5.8	7.9

3. In writing decimal tenths, you wrote the decimal point and only the numerators of the fractions. How did you know that .5 means 5 tenths? That 5.8 means 5 and 8 tenths?

4. The first place to the right of the decimal point is the _?_ place.

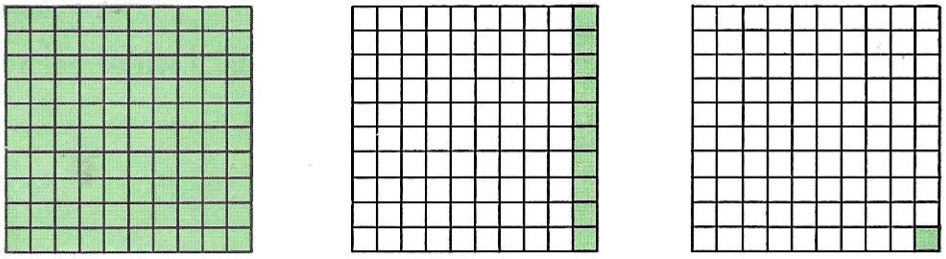
Seventeen hundredths may be written as the fraction $\frac{17}{100}$ or as the decimal .17. One hundredth may be written as the fraction $\frac{1}{100}$ or as the decimal .01.

5. $\frac{17}{100}$ means 17 hundredths, so .17 means _?_.
   $\frac{1}{100}$ means _?_, so .01 means _?_.

6. How many places to the right of the decimal point are used to write hundredths? In writing 1 hundredth as a decimal, why is the zero necessary?

7. Copy the following table leaving blanks where there are question marks. Then fill in the blanks so that each column will show two ways of writing the same number. Read each number. Remember to say *and* for the decimal point when you read a mixed decimal.

fraction	$\frac{13}{100}$	$\frac{4}{100}$	$\frac{9}{100}$	?	?	$1\frac{23}{100}$	$1\frac{7}{100}$	$1\frac{1}{100}$	?	?
decimal	?	.04	?	.27	.03	?	1.07	?	2.37	5.09



Study the diagram above. Explain these statements:

8. The colored part of the first square represents 1 whole. It may also represent 10 tenths, 1.0. It may also represent 100 hundredths, 1.00.

9. The colored part of the second square represents 1 tenth, .1. It may also represent 10 hundredths, .10.

10. The colored part of the third square represents 1 hundredth, .01.

11. If you put together the three colored parts, you get 1 whole 1 tenth 1 hundredth. This number may be written 1.11.

Thinking about Decimals

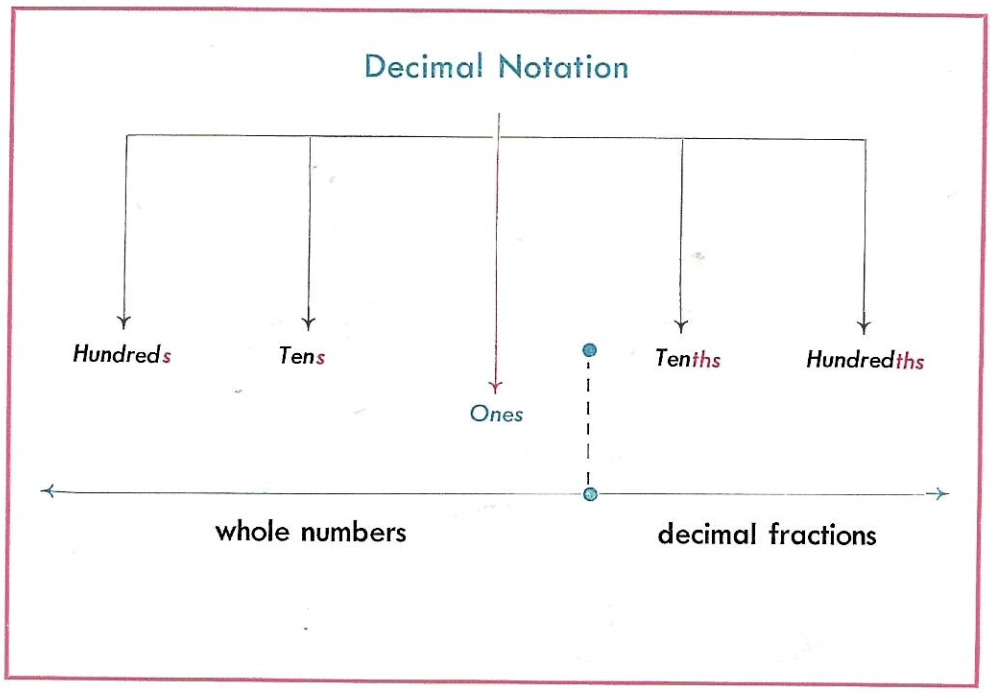
Explain the following statements. Use the diagram on page 159 for help if you need it.

1. Each figure in 1.11 stands for a different size. Its size is shown by its place in the numeral.
2. Each 1 in 1.11 is 10 times as large as the 1 to its right.
3. Each 1 in 1.11 is $\frac{1}{10}$ as large as the 1 to its left.

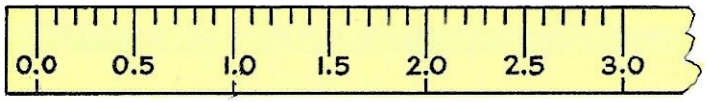
The word *decimal* comes from the Latin word which means ten. Our system of numeration is called a decimal system because it is based on ten. The value of each place in our system is ten times the value of the place next to it on the right, or one-tenth the value of the place next to it on the left. This is true whether the places are before or after the decimal point.

Study the following chart.

✪ See if you can copy it and make it one place wider each way.



4. Each inch on the ruler shown below is divided into ten equal parts. Each small line marks .1 inch. Point to the line marked 0.0. Measures start at 0.0 on this ruler. From 0.0 to the line marked 1.0 is 1 inch. 1.0 means 1 whole and 0 tenths. The 0 shows that the measurement is made in tenths of an inch. What does 2.0 mean? 3.0?



5. Point to the 0.5-inch line. What does this 0 mean? Measures smaller than 1 inch are often written with zero in ones place. With the zero there, the reader is more likely to see the decimal point and read the number correctly. Is this zero necessary? Would the number mean the same without it? Point to the line marking 0.1 inch; 0.2 inch; 0.3 inch; 0.4 inch; 0.6 inch; 0.9 inch.

6. Find on the ruler above the lines marking 1.5 inches; 1.3 inches; 1.7 inches; 2.2 inches; 2.5 inches; 2.8 inches.

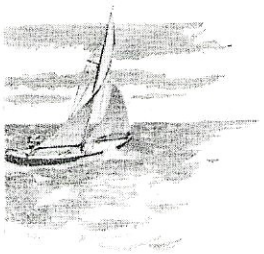
Using the ruler to help, if necessary, tell the missing numbers.

7. .3, .4, .5, .6, ?, ?, ?, 1.0, ?, 1.2.
8. 1.7, 1.8, ?, 2.0, ?, ?, ?, 2.4, ?, ?.
9. .2, .4, .6, ?, 1.0, 1.2, ?, ?, ?, ?.
10. .3, .6, ?, 1.2, ?, ?, ?, ?, ?, ?, ?.
11. 1.0, .9, .8, ?, ?, ?, ?, ?, ?, ?, ?.

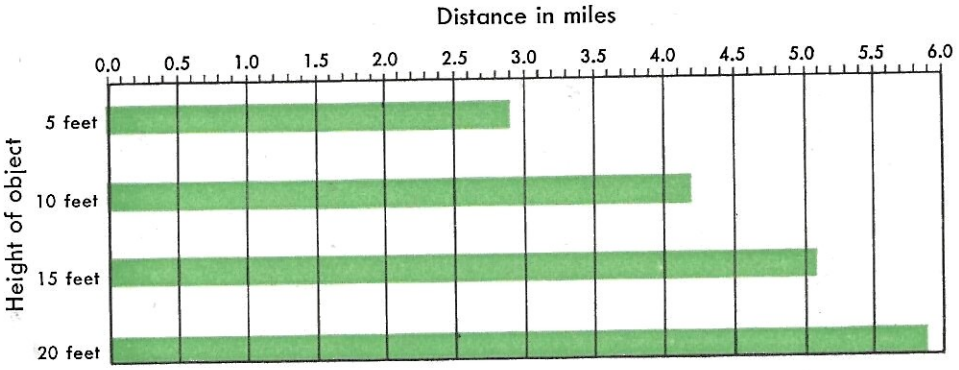
One hundredth of an inch is very small. Imagine each tenth on the ruler above being divided into ten equal parts. Then each small part would be .01 inch. A very fine hair is about .01 inch thick.

Tell the missing numbers.

12. .01, .02, .03, ?, ?, ?, ?, ?, ?, ?, ?.
13. .02, .04, ?, ?, .10, ?, ?, ?, ?, .20.



DISTANCE OF VISIBILITY



Decimals in a Graph

Sandy lay on the shore watching a boat sail away into the distance. He wondered how far away the boat was when it disappeared from view. If Sandy knew the height above water of the boat's tallest mast, he could find out about how far away the boat was.

The graph above shows about how many miles an object of given height can be seen by a person whose eye is at water level. Read the title of the graph. What do the numbers along the left side tell you? The scale of miles across the top uses mixed decimals, like the ruler on the previous page.

1. How many tenths of a mile does each small space on the scale at the top of the graph represent? How many tenths are there in a mile?
2. What is the approximate distance in miles an object 5 feet high can be seen on a lake? an object 10 feet high? an object 15 feet high? an object 20 feet high?
3. The higher an object on a lake, the _?_ distance it can be seen.
4. Which is larger, 0.5 or 1.0? 1.1 or 0.9? 2.1 or 2.5? 2.5 or 1.8? 3.0 or 2.7? 5.0 or 4.9? 0.6 or 0.7?

   Use the scale at the top of the graph if you need help.

Adding and Subtracting Tenths

NO REGROUPING

Tenths are added and subtracted just as tens or ones or any like numbers are added and subtracted. Study these examples.

Add. Remember that 10 ones = 1 ten and that 10 tenths = 1 one.

3 ones	3	3 tenths	.3	9 ones	9	9 tenths	.9
2 ones	2	2 tenths	.2	4 ones	4	4 tenths	.4
5 ones	5	5 tenths	.5	13 ones	13	13 tenths	1.3

Subtract. Remember that 1 ten = 10 ones and that 1 one = 10 tenths.

9 ones	9	9 tenths	.9	11 ones	11	11 tenths	1.1
7 ones	7	7 tenths	.7	8 ones	8	8 tenths	.8
2 ones	2	2 tenths	.2	3 ones	3	3 tenths	.3

1. What can you say about the position of the decimal points in addition and subtraction examples?

Copy and add. Keep the decimal points in line so that tenths will be under tenths and ones under ones.

2.	.1	.2	.3	.6	.4	.5	.7	.3	.5
	.3	.4	.5	.2	.5	.1	.2	.6	.5
3.	.3	.5	.6	.7	.8	.9	.8	.7	.9
	.8	.7	.6	.8	.8	.9	.6	.4	.8

Copy and subtract. Keep the decimal points in line.

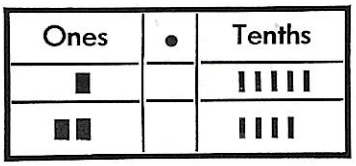
4.	.9	.8	.7	.6	.5	.9	.4	.8	.2
	.8	.6	.3	.2	.1	.4	.2	.7	.2
5.	1.1	1.1	1.2	1.0	1.2	1.0	1.3	1.4	1.5
	.1	.2	.3	.5	.7	.6	.9	.7	.2
6.	1.3	1.4	1.5	1.6	1.7	1.8	1.6	1.7	1.8
	.4	.8	.5	.7	.8	.9	.9	.7	.5

Adding Mixed Decimals

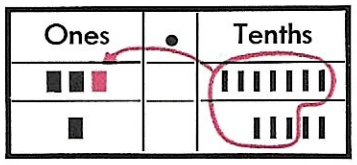
WITH AND WITHOUT REGROUPING

A place-value chart will help you see why we use the same kind of thinking in working with decimals that we use in working with whole numbers.

Study the examples below. Explain how we record in figures the additions shown in each picture of a place chart. Why is it not necessary to regroup the numbers in the first example? Why must we regroup the numbers in the second example?



$$\begin{array}{r} 1.5 \\ +2.4 \\ \hline 3.9 \end{array}$$



$$\begin{array}{r} {\scriptstyle 1} \\ 2.7 \\ +1.5 \\ \hline 4.2 \end{array}$$

Find the sums. Add down. Check by adding up.

1.	2.4	2.7	7.0	.8	7.6
	4.3	6.1	9.5	3.1	9.2
2.	99.1	.3	31.8	4.0	70.5
	90.4	.6	2.0	7.8	8.4
3.	4.6	12.7	20.8	.4	298.3
	3.5	1.6	45.9	.9	21.8
4.	29.6	2.7	14.8	82.3	52.6
	82.6	19.3	85.2	14.8	99.5

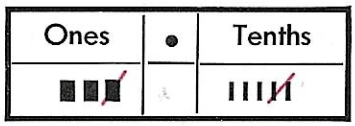
Arrange in columns. Be sure to keep the decimal points in line. Add down. Check by adding up.

5. $1.6 + 6.2$	$1.4 + 3.1$	$8.0 + 1.2$
6. $2.8 + 3.7$	$5.6 + 7.5$	$7.9 + 7.9$
7. $20.6 + 37.8 + 14.7$		$4.9 + 25.6 + 131.8$
8. $2.3 + 14.8 + 7.5$		$2.6 + 9.0 + 34.9$

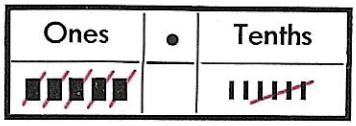
Subtracting Mixed Decimals

WITH AND WITHOUT REGROUPING

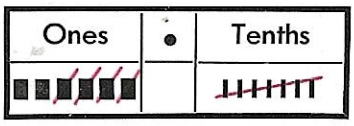
Study the examples below. Explain how we record in figures the subtractions shown in each picture of a place-value chart. Why must we regroup the numbers in the minuend of the last example?



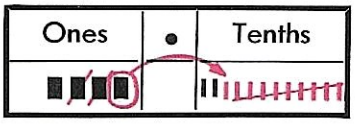
$$\begin{array}{r} 3.5 \\ -1.2 \\ \hline 2.3 \end{array}$$



$$\begin{array}{r} 5.6 \\ -5.4 \\ \hline .2 \end{array}$$



$$\begin{array}{r} 6.7 \\ -4.7 \\ \hline 2.0 \end{array}$$



$$\begin{array}{r} {}^{3}\cancel{4}.{}^{12}2 \\ -2.9 \\ \hline 1.3 \end{array}$$

Copy and subtract. Check by adding.

1.	8.5	6.2	9.6	12.7	7.4
	4.3	4.0	4.6	5.6	.4
2.	6.8	7.9	24.7	54.5	39.6
	6.2	2.0	1.7	24.3	15.0
3.	5.1	8.3	3.7	4.2	7.4
	2.4	3.8	.9	.8	6.6
4.	5.0	12.0	10.5	13.6	15.0
	3.1	8.6	2.7	12.8	7.3

Arrange in columns. Be sure to keep the decimal points in line. Subtract. Check by adding.

5.	23.8 − 12.6	35.7 − 4.3	9.5 − .5
6.	8.7 − 4	27.6 − 13	15.3 − 5.3
7.	7.7 − 3.9	12.3 − 9.8	13 − 6.2
8.	3.2 − .9	20.4 − 13.8	24 − 14.1
9.	31.3 − 11.5	44.5 − 23.6	36 − 18.7

Getting to the Point

1. Joe wrote: "I spent $19.8 for a baseball bat." Is Joe's statement sensible? What do you think he meant to write?
2. Jane asked her mother for money to buy a chocolate bar. Do you think her mother gave her $5.00, $.50, $.05, or $.005?

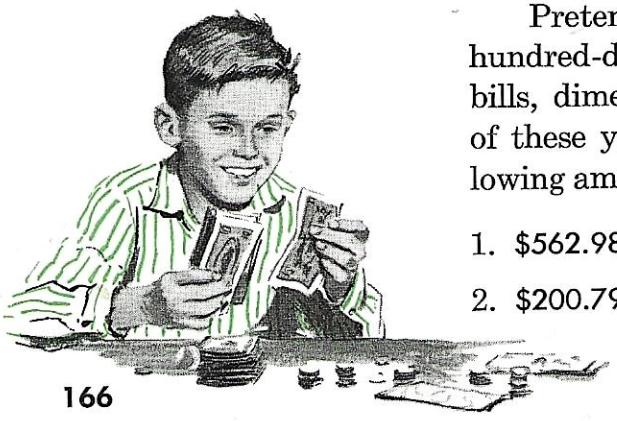
It is a serious mistake to leave out a decimal point in an answer or to put it in the wrong place. Answers must make sense.

Copy the sentences below. Write a decimal point in each number so that the statements will be sensible.

3. The little brook in our back yard is 29 feet wide.
4. I had to pay $23 out of every dollar for taxes.
5. My tennis racket is 15 inches longer than Jim's.
6. The car that won the race traveled at an average speed of 1218 miles per hour.
7. Jack is in sixth grade. He says he is 575 inches tall.
8. David says he can jump over a bar 40 feet high.
9. Tom made a profit of $149 on each turkey he raised last year.
10. Our dollar bills are 614 inches long and 261 inches wide.

Decimals and United States Money

ADDING AND SUBTRACTING HUNDREDTHS



Pretend that you have a supply of money in hundred-dollar bills, ten-dollar bills, one-dollar bills, dimes, and pennies. Tell how many of each of these you would need to make each of the following amounts of money.

1.	$562.98	$43.16	$3.57	$423.08
2.	$200.79	$10.01	$600.30	$603.03

Our system of writing money numbers is a decimal system because it is based on ten.

3. Each 1 in \$111.11 is worth a different amount. Tell how much each 1 is worth.

Tell why the following statements are true.

4. Each 1 in \$111.11 is worth 10 times as much as the next 1 to its right.

5. Each 1 in \$111.11 is worth $\frac{1}{10}$ as much as the next 1 to its left.

6. \$.01 may stand for 1 cent. A cent is 1 hundredth of a dollar. \$.01 really means 1 hundredth of a dollar.

7. \$.10 may stand for 1 dime. A dime is 1 tenth of a dollar. It is also worth 10 cents or 10 hundredths of a dollar. \$.10 really means 10 hundredths of a dollar.

8. \$.25 may mean 25 cents or 25 hundredths of a dollar. It may also mean 2 tenths 5 hundredths of a dollar.

Since the two figures to the right of the decimal point in money numbers stand for tenths and hundredths of a dollar, you can add and subtract decimals and mixed decimals with tenths and hundredths just as you add and subtract dollars and cents.

Copy and add.

9.	\$.56	\$1.14	\$8.98	\$14.50	\$207.13
	.27	5.79	9.89	29.50	49.98
10.	.56	1.14	8.98	14.50	207.13
	.27	5.79	9.89	29.50	49.98

Copy and subtract.

11.	\$.79	\$2.50	\$7.75	\$10.00	\$506.14
	.24	1.29	7.50	4.39	490.15
12.	.79	2.50	7.75	10.00	506.14
	.24	1.29	7.50	4.39	490.15

Practice to Remember

MAINTENANCE TEST ON MEANINGS AND SKILLS (CHAPTERS 1–4)

1. Round each of these numbers to the nearest hundred.

   478 721 943 690 799

2. Write in figures: Two million, three hundred four thousand, six hundred fifty.

3. Multiply 264 by 5. Then multiply 264 by 60. Add the two products. Is the sum the same as the product of 65 and 264? Multiply 264 by 65 to see.

4. Find the perimeter of a rectangle 37 ft. by 45 ft.

5. Find the area of a rectangle 18 in. by 36 in.

6. 3 gal. = ? qt. 56 oz. = ? lb. 27 sq. ft. = ? sq. yd.

Copy. Multiply. Check your work.

7. $\begin{array}{r} 76 \\ \times\ 43 \\ \hline \end{array}$ $\begin{array}{r} 467 \\ \times\ 56 \\ \hline \end{array}$ $\begin{array}{r} 236 \\ \times\ 34 \\ \hline \end{array}$ $\begin{array}{r} 374 \\ \times\ 83 \\ \hline \end{array}$ $\begin{array}{r} 789 \\ \times\ 40 \\ \hline \end{array}$

8. $\begin{array}{r} \$7.50 \\ \times\ 45 \\ \hline \end{array}$ $\begin{array}{r} \$15.98 \\ \times\ 7 \\ \hline \end{array}$ $\begin{array}{r} \$16.55 \\ \times\ 127 \\ \hline \end{array}$ $\begin{array}{r} 278 \\ \times\ 406 \\ \hline \end{array}$ $\begin{array}{r} 475 \\ \times\ 120 \\ \hline \end{array}$

Copy. Divide. Check your work.

9. $8\overline{)1632}$ $7\overline{)1429}$ $40\overline{)\$1.60}$ $60\overline{)180}$ $23\overline{)483}$

10. $34\overline{)1794}$ $61\overline{)2512}$ $13\overline{)\$2.73}$ $13\overline{)273}$ $51\overline{)2771}$

Do as the signs and words tell you.

11. $\frac{1}{5}+\frac{3}{5}$ $\frac{4}{7}+\frac{5}{7}$ $2\frac{1}{4}+3\frac{1}{4}$ $\frac{1}{2}+\frac{1}{6}$ $\frac{1}{5}+\frac{2}{3}$

12. $\frac{4}{5}-\frac{1}{5}$ $\frac{5}{8}-\frac{1}{8}$ $6\frac{2}{3}-2\frac{1}{3}$ $\frac{7}{8}-\frac{1}{4}$ $4\frac{3}{4}-1\frac{1}{3}$

13. $\frac{1}{2}$ of 12 $\frac{2}{3}$ of 18 $\frac{3}{4}$ of 20 $\frac{1}{3}$ of 36 $\frac{1}{4}$ of 48

14. $\frac{1}{5}$ of 30 $\frac{4}{5}$ of 30 $\frac{1}{9}$ of 27 $\frac{5}{9}$ of 27 $\frac{3}{4}$ of 48

Practice to Remember

MAINTENANCE TEST ON PROBLEM SOLVING (CHAPTERS 1–4)

Read each problem carefully. Decide whether to add, subtract, multiply, or divide. Then find the answer.

1. Last month Arthur earned $5.20 and Gordon earned $3.85. How much more did Arthur earn than Gordon?
2. During last year, Mr. Jackson traveled 8,732 miles by plane, 4,379 miles in his car, and 1,307 miles by train. What was the total number of miles he traveled last year?
3. On a two-day trip, Mr. Calder drove his car 532 miles in 14 hours. How many miles an hour did he average?
4. For the School Fair, Judy put 150 cookies in bags with 6 cookies in each bag. How many bags did she use?
5. Mrs. Long bought 5 pounds of sugar. She has $3\frac{1}{2}$ pounds left. How many pounds were used?
6. What is the total weight of three lobsters weighing $3\frac{1}{2}$ pounds, $2\frac{1}{8}$ pounds, and $1\frac{3}{4}$ pounds?
7. Joe began to deliver newspapers at 4:12 P.M. He finished at 4:48 P.M. How long did he work?
8. Mr. Brown wants to build a wire fence around his garden. The garden is a square 38 feet on each side. How much wire fence will he need?
9. How many square feet of carpet will be needed to cover the floor of a room 14 feet by 23 feet?
10. Four boys weigh as follows: 103 pounds, 114 pounds, 98 pounds, and 105 pounds. What is the average weight of these boys?
11. The Girl Scouts bought 55 boxes of cookies at 35 cents a box. They sold all the cookies for $27.50. How much money did they make?

Thousandths

MEANING, READING, WRITING

Jane read in a science book that if her hair is of medium texture, one hair is about .022 inch thick. Can you read this number?

You know that the first place to the right of the decimal point is tenths place, and that the second place to the right of the decimal point is hundredths place. The third place to the right of the decimal point is thousandths place. Another way to write .022 is $\frac{22}{1000}$, and both are read 22 *thousandths*.

In reading a decimal, read the number as if it were a whole number, then say the name of the place of the last figure at the right.

.6 is read 6 tenths.
.04 is read 4 hundredths.
.37 is read 37 hundredths.
.001 is read 1 thousandth.
.049 is read 49 thousandths.
.285 is read 285 thousandths.

Read these numbers.

1. .467 .049 .568 .011
2. .403 .014 .009 .808

Write as decimals:

3. $\frac{341}{1000}$ $\frac{209}{1000}$ $\frac{38}{1000}$ $\frac{22}{1000}$
4. $\frac{3}{1000}$ $\frac{8}{1000}$ $\frac{7}{1000}$ $\frac{5}{1000}$
5. 114 thousandths; 439 thousandths
6. 77 thousandths; 29 thousandths
7. 6 thousandths; 9 thousandths

You know that .1 is 10 times as large as .01. As you would expect, .01 is 10 times as large as .001. If we tried to show .001 on a square such as we used to show .1 and .01, each thousandth would be so small that it would be hard to see.

An ordinary sheet of paper is .025 inch thick. If you could split a piece of paper into 25 thinner sheets, each of these would be .001 inch thick. You can see that it would be impossible to show .001 inch on a ruler. In making parts and instruments for airplanes, automobiles, and other machines, very fine measurements are necessary, measurements to thousandths of an inch and even smaller units. A *micrometer* is used to make these measurements in very small units.

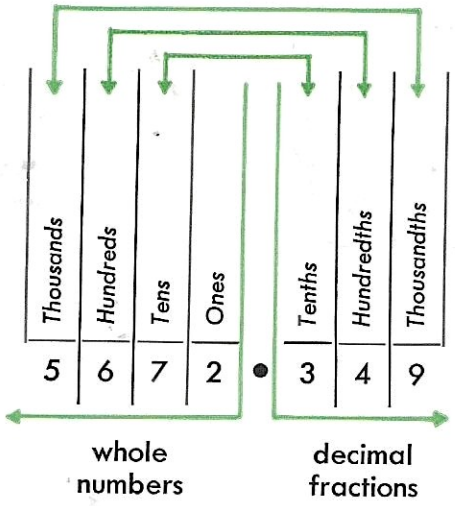
8. See if you can find out how a micrometer works. Perhaps someone you know has a micrometer and will let you bring it to school to show your class.

Something Old, Something New

PLACE VALUE; MILLS

Study the place-value chart below.

1. Read the number in the chart. Say *and* for the decimal point.



2. Notice the arrows with the chart. What do they show? If you know the names of the places to the left of ones place, how can you tell the names of the places to the right of ones place? Remember that the names of places for whole numbers end in *s* and that the names of places for decimal fractions end in *ths*.

There is a division of United States money called a *mill*. There is no coin with a value of a mill, but some states have issued tokens valued in mills for people to use in paying sales taxes. A mill is 1 thousandth of a dollar. A mill is 1 tenth of a cent. One mill is written \$.001.

Study the table below.

1 dollar	=	\$1.00
1 dime	=	.10
1 cent	=	.01
1 mill	=	.001
Sum	=	\$1.111

3. Complete: One dollar is 10 times 1 dime. One dime is 10 times _?_. One cent is 10 times _?_. One mill is $\frac{1}{10}$ of 1 _?_. It takes _?_ mills to make 1 cent.

4. How many of \$1000, \$100, \$10, and \$1 bills and how many dimes, cents, and mills are needed to make up the following sums?

\$3674.219	\$14.016	\$5.005
243.532	7.308	6.210

5. Is the rule below true for money numbers?

The value of each place in a whole number, decimal, or mixed decimal is 10 times the value of the next place to the right, and $\frac{1}{10}$ the value of the next place to the left.

Practice in Adding and Subtracting Decimals

Decimals fit neatly into our place-value system, so we add and subtract decimals and mixed decimals just as we add and subtract whole numbers.

In copying each example below, be sure to keep the decimal points in line. Then the decimal point in your answer will come directly under the other decimal points in the example.

Find the sums. Add down. Check by adding up.

1.	36.82	2.42	72.09	84.35	.87
	51.14	3.36	64.13	97.26	.08
2.	14.34	3.56	87.48	22.93	.86
	.56	4.27	22.54	43.25	.99
	2.06	10.45	415.96	.43	.07
3.	.719	4.213	3.518	7.236	.764
	.388	5.684	6.273	4.201	.007
	.139	.354	14.308	.005	.450
	.432	6.006	.451	8.000	.307

Arrange in columns and add. Be sure to keep the decimal points in line.

4. $28.18 + 9.09 + .80 + 24.03$ $7.241 + 6.329 + .076$
5. $32.45 + .47 + .07 + 3.40$ $.564 + .370 + 4.539$
6. $2.732 + 4.203 + .371 + 1.025$ $32.45 + .07 + 3.40$
7. $.731 + .426 + .120 + .036$ $8 + .32 + 7.04$

Copy and subtract. Check by adding.

8.	56.7	7.94	78.6	62.5	9.436
	31.5	3.62	45.5	11.5	8.423
9.	12.475	.78	.649	42.72	78.843
	8.372	.71	.642	22.52	45.443

10.	.86 .42	.768 .654	.893 .691	.457 .432	.33 .02
11.	.67 .62	.479 .475	.457 .423	.643 .343	.54 .50
12.	15.4 11.6	7.81 5.34	8.91 4.27	9.423 2.118	34.412 21.305
13.	1.5 .7	15.4 8.6	.62 .45	6.324 2.147	18.712 12.856
14.	703.4 358.9	.837 .459	46.80 24.46	8.324 2.375	62.198 19.309
15.	3.54 2.49	38.2 24.5	12.836 9.782	6.341 6.174	28.32 27.68

Arrange in columns and subtract. Check by adding.

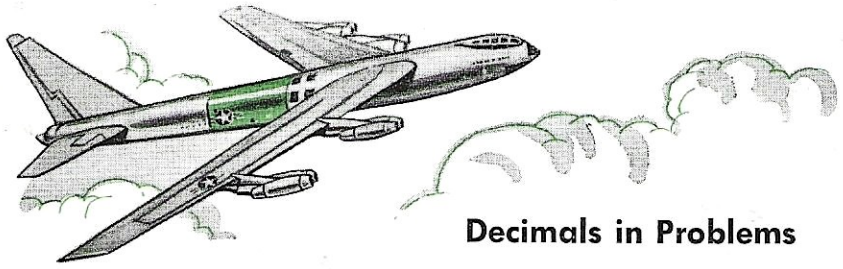
16.	.9 — .4	27.56 — 14.34	6.547 — 3.236
17.	.8 — .2	35.59 — 32.39	15.538 — 7.438
18.	1.3 — .2	67.54 — 43.51	47.645 — 7.643
19.	13.5 — .5	16.32 — 6.12	35.465 — 5.365
20.	\$16.00 — \$5.86	\$64.25 — \$32.75	\$112.05 — \$98.76
21.	27.34 — 14.56	9.324 — 6.318	.987 — .974
22.	121.34 — 62.47	28.314 — 14.127	6.342 — 6.227

23. Write the following numbers in the order of their size. Begin with the smallest.

.2 1.4 .6 .7 .5 .9 .3 1.0 1.7 2.4 1.9

24. Using your answer to example 23, find the difference of the first and second numbers, the second and third, and so on.

25. Find the sum of all the numbers in example 23.



Decimals in Problems

Read each problem carefully. Think what it asks you to find. Decide whether to add or subtract to find the answer. The first two problems have hints to help you tell what a sensible answer would be. For each problem, make a rough estimate of a sensible answer, then solve the problem. If your answer is not sensible, go over all your work.

1. In a recent airplane race across the nation, the winner in one class flew 507.255 miles per hour. The winner in another class flew 460.423 miles per hour. How much faster did one winner fly than the other?

   Hint: Should the answer be less than 100 miles per hour? About 50 miles per hour?

2. A commercial airplane flew 196.6 miles in the first hour, 203.7 miles in the second hour, 193.8 miles in the third hour, and 211.3 miles in the fourth hour. How far did it fly in the four hours?

   Hint: Each number is about 200, so the answer should be about ?.

3. A machinist measured two bars of metal. One was 17.53 inches long. The other was 12.65 inches long. How much longer was one bar than the other?

4. In a city a weather bureau reported the following amounts of rainfall by weeks for a month: 1.18 inches, 2.66 inches, 1.67 inches, and 2.95 inches. How much rain fell during the month?

5. On an automobile trip Mr. King traveled 227.7 miles the first day and 324.4 miles the second day. How many miles did he travel in the two days?

6. A sign at a gasoline station read, "Gas 15.4¢, federal tax 3.0¢, state tax 5.5¢." What was the price per gallon of gasoline?

7. On May 12, 1951, Mrs. Jacqueline Auriol of France flew a jet plane at an official record average speed of 509.245 miles per hour. Miss Jacqueline Cochran of the United States held the previous world speed record for women, 469.549 miles per hour. How much faster was Mrs. Auriol's speed than Miss Cochran's?

8. Whether we feel warm or cold, the blood is kept at a normal temperature of about 98.6° Fahrenheit at all times. When Tommy was sick, his temperature was 103.4°. How much above normal was Tommy's temperature?

9. Mr. Stone measured the lot in which he will keep his horse. He found that the four sides measured 51.5 ft., 76.7 ft., 50.9 ft., and 75.4 ft. How much fencing will he need to enclose the lot?

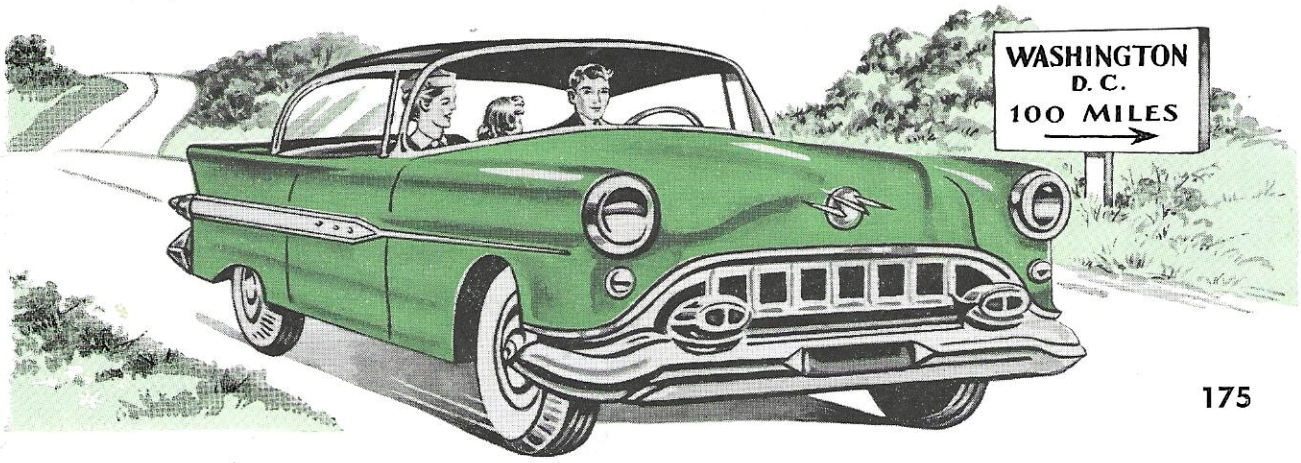
10. Walter went shopping for his mother. He paid $3.02 for meat, $2.37 for groceries, and $.98 for shampoo. How much should he have left out of $10.00?

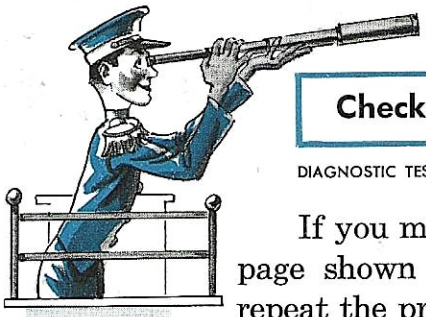
11. At a crossroads, a sign read, "Road ends 1.3 mi." Jerry checked the sign with his bicycle cyclometer. At the sign, his cyclometer read 18.9 miles. At the end of the road, his cyclometer read 20.1 miles. What was the difference between Jerry's measure and the distance given on the road sign?

12. In June, Tony weighed 68.5 pounds and in September he weighed 71.3 pounds. Larry weighed 70.6 pounds in June and 73.0 pounds in September. Which boy gained more weight during the summer? How much more?

13. In a Southern state gasoline sells for 27.5¢ per gallon, including taxes. The state tax is 6.0¢ per gallon and the federal tax is 3.0¢ per gallon. What would be the selling price of the gasoline if there were no taxes?

14. When the Smith family left Richmond, Virginia, the speedometer on the car read 23176.3 miles. A sign said, "Washington, D. C., 100 mi." When the speedometer read 23198.8 miles, Sue asked, "How much farther is it to Washington?" Answer Sue's question.





Checking Up on Decimals

DIAGNOSTIC TEST ON PP. 158–172

If you make mistakes in a row of examples, turn to the page shown beside the row. Study the explanation and repeat the practice. Then try the row on this page again.

158 1. Write as decimal fractions or mixed decimals.

$\frac{6}{10}$ $\frac{9}{10}$ $\frac{15}{100}$ $8\frac{4}{10}$ $12\frac{35}{100}$ $9\frac{6}{100}$

158 2. Write as common fractions or mixed numbers.

.8 .5 .27 8.7 29.2 47.03

Copy and add. Keep the decimal points in line.

163 3.

.2	.4	.7	.9	.6	.4
.3	.5	.1	.5	.9	.6

164 4.

1.2	6.0	5.6	9.1	76.3	9.1
7.3	2.5	9.8	0.7	8.0	18.9

Copy and subtract. Check by adding.

163 5.

.9	.3	.7	1.5	1.1	1.6
.7	.1	.6	.9	.3	.6

165 6.

7.6	9.8	3.4	3.2	8.5	5.0
2.1	6.8	2.0	.7	7.8	1.9

Arrange in columns and add. Be sure to keep the decimal points in line.

164 7. 5.2 + 3.7 1.6 + 2.9 8.0 + 22.6

167 8. \$4.25 + \$3.50 \$5.08 + \$.96 3.10 + 7.39

Arrange in columns and subtract.

165 9. 3.6 − 1.2 7.0 − 5.4 9.1 − .8

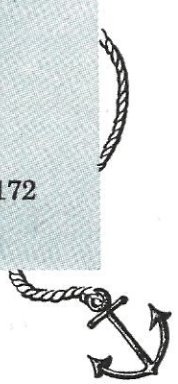
167 10. \$2.75 − \$1.10 \$5.19 − \$4.89 7.00 − 5.05

Copy and add. Check by going over your work.

11.	$7.86	3.91	4.321	6.978	.798	172
	.50	8.70	9.006	.540	.563	
	2.01	1.00	3.654	4.039	.102	

Copy and subtract. Check by adding.

12.	$4.89	6.41	4.846	9.750	5.000	172
	3.50	1.18	2.105	1.986	1.375	



Changing Decimals to Common Fractions

It is easy to change a decimal into a common fraction. Read the decimal to yourself, then write as a common fraction what you have said.

.3 is read "3 tenths." "Tenths" tells you that the denominator is 10. As a common fraction .3 is written $\frac{3}{10}$.

.03 is read "3 hundredths." What is the denominator of this fraction? Written as a common fraction .03 is $\frac{3}{100}$.

.003 is read "3 thousandths." Written as a common fraction .003 is $\frac{3}{1000}$. Why?

Write the following decimals as common fractions with denominators of 10, 100, or 1000.

1.	.1	.3	.7	.5	.6
2.	.03	.07	.09	.11	.21
3.	.25	.20	.50	.75	.90
4.	.003	.007	.009	.005	.001
5.	.025	.325	.050	.225	.600
6.	.8	.33	.10	.008	.750

7. How can you tell whether a fraction is in lowest terms? What do you do to change a fraction to lowest terms?

8. Change the following fractions to lowest terms.

$\frac{2}{10}$, $\frac{5}{10}$, $\frac{8}{10}$, $\frac{10}{100}$, $\frac{20}{100}$, $\frac{25}{100}$, $\frac{50}{100}$, $\frac{500}{1000}$.

To change a decimal to a common fraction in lowest terms, write the decimal as a common fraction with a denominator of 10, 100, or 1000; then change the fraction to lowest terms.

Change the following decimals to common fractions in lowest terms.

9.	.2	.4	.5	.6	.8
10.	.40	.30	.60	.70	.80
11.	.75	.25	.250	.300	.800

Changing Common Fractions to Decimal Form

Since common fractions and decimals are two ways of expressing the same numbers, fractions may be changed to decimals and decimals may be changed to fractions having the same value. In some examples, decimals are easier to work with than fractions. In other cases, fractions are easier to work with than decimals. Because of this, it is desirable to know how to change one to the other. It is easy to do this if you think about the meaning of the numbers.

To change $\frac{1}{5}$ to a decimal, think: Decimals have unwritten denominators of 10, 100, 1000, and the like. I must change the denominator of $\frac{1}{5}$ to 10, 100, or 1000, and change the numerator so that the new fraction will have the same value as $\frac{1}{5}$. Then I can write it as a decimal.

1. Can fifths be changed to tenths? $\_?\_ \times 5 = 10$. Explain the following: $\frac{1}{5} = \frac{2 \times 1}{2 \times 5} = \frac{2}{10}$. Then $\frac{1}{5} = .2$.

2. Change the following fractions to decimal tenths.

$$\frac{1}{5} \qquad \frac{2}{5} \qquad \frac{3}{5} \qquad \frac{4}{5} \qquad \frac{1}{2}$$

3. Is there a whole number by which you can multiply 4 to get 10? Can fourths be changed to tenths, then? Is there a whole number by which you can multiply 4 to get 100? $\_?\_ \times 4 = 100$?

$$\frac{1}{4} = \frac{25 \times 1}{25 \times 4} = \_?\_ \qquad \frac{1}{4} = \frac{25}{100} = .25$$

4. Change the following fractions to decimal hundredths.

$$\frac{1}{4} \qquad \frac{3}{4} \qquad \frac{1}{2} \qquad \frac{2}{5} \qquad \frac{3}{5}$$

5. By what number would you multiply each of the following to get 1000? 2, 4, 8, 5, 10

6. Use your answer to example 5 to change the following fractions to decimal thousandths.

$$\frac{1}{2} \qquad \frac{1}{4} \qquad \frac{1}{8} \qquad \frac{1}{5} \qquad \frac{3}{10}$$

Comparing Decimals

In comparing decimals, you have to think about place value and about the size of the decimal parts.

Study the facts below. Read across the page.

Common Fractions	Decimals
$\frac{1}{10}$ has 1 zero in the denominator.	.1 has 1 decimal place.
$\frac{1}{100}$ has 2 zeros in the denominator.	.01 has 2 decimal places.
$\frac{1}{1000}$ has 3 zeros in the denominator.	.001 has 3 decimal places.

There are as many places to the right of a decimal point in a decimal fraction as there are zeros in the denominator of the common fraction that means the same.

1. What is the largest one-place decimal that you can write? The smallest? The largest two-place decimal? The smallest? The largest three-place decimal? The smallest?

2. Which is larger: \$.25 or \$.38? .05 or .09? .16 or .10? .37 or .30? .05 or .02? .09 or .90? 3.45 or 2.48? 12.63 or 14.06? 27.07 or 16.31?

3. Write the following decimals in the order of their size. Begin with the smallest.

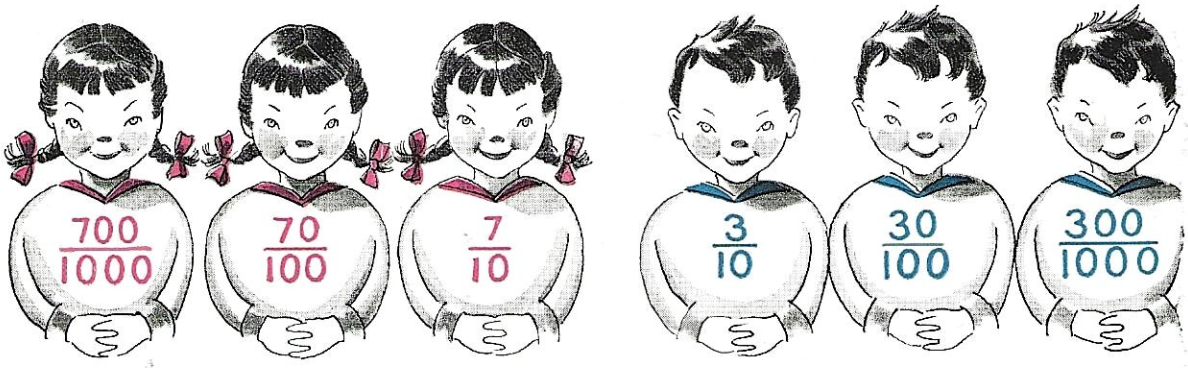
   .11 .05 .26 .57 .38 .74 .90

4. Which is larger: .125 or .225? .347 or .298? .002 or .005? .017 or .029? .038 or .019? .304 or .413? 6.702 or 5.968? 21.037 or 21.007? 3.014 or 2.789?

5. Write the following batting averages of several members of a baseball team in order of size from highest to lowest.

   .199 .302 .201 .137 .319 .298

6. How can you tell which of two decimal fractions is larger when they have the same number of decimal places?



Changing Decimals to Lower or Higher Terms

Fractions written as decimals may be changed to lower or higher terms just as fractions written as common fractions may be changed to lower or higher terms.

$$\frac{700}{1000} = \frac{700 \div 10}{1000 \div 10} = \frac{70}{100} \qquad \frac{70}{100} = \frac{70 \div 10}{100 \div 10} = \frac{7}{10}$$

The examples above show that:

700 thousandths	= 70 hundredths	= 7 tenths
.700	= .70	= .7

1. How would you change the decimal .700 to hundredths? To tenths?

$$\frac{3}{10} = \frac{10 \times 3}{10 \times 10} = \frac{30}{100} \qquad \frac{30}{100} = \frac{10 \times 30}{10 \times 100} = \frac{300}{1000}$$

The examples above show that:

.3 tenths	= 30 hundredths	= 300 thousandths
.3	= .30	= .300

2. How would you change the decimal .3 to hundredths? To thousandths?

Zeros written after the last figure to the right of a decimal point change the terms but not the value of the decimal.

Copy the following headings and decimals. Complete by changing the terms of the decimal given.

	Tenths	*Hundredths*	*Thousandths*
3.	.3	.30	.300
4.	.6	?	?
5.	?	.80	?
6.	?	?	.400
7.	1.7	?	?

To compare decimals of different terms, change the decimals to the same terms by writing zeros when necessary.

For example, to compare .3 and .28, rewrite .3 as .30. Then compare. .3 is larger than .28 because .30 is larger than .28.

.46 is smaller than .519 because .460 is smaller than .519.

8. Which is largest: .7, .72, or .726? Which is smallest?
9. Which is largest: .07, .17, or .71? Which is smallest?
10. Which decimals have the same value: .04 .4 .40 .400?
11. Arrange in order from smallest to largest: .29 .195 .5 2.9

Just for Fun

In a magic square the sum of the numbers in each row, each column, and each diagonal (from corner to corner) is the same.

1. Copy Square A below. How many rows are there? How many columns? How many diagonals? How many addition examples would you have to do to find out whether this is a magic square? Is Square A a magic square?
2. In the same way find out whether Square B is a magic square.
3. Copy Square C leaving blank spaces in place of the question marks. Then fill in the blank spaces so that Square C will be a magic square.

.8	.1	.6
.3	.5	.7
.4	.9	.2

A

.7	.5	.3
.2	.9	.4
.6	.1	.8

B

?	1.8	.4
.6	1.0	?
1.6	?	1.2

C

What Have You Learned about Decimals?

CHAPTER TEST ON MEANINGS AND SKILLS

1. Write *seven tenths* as a common fraction; as a decimal fraction.

2. Write the words which complete this sentence.
   To the right of the decimal point, the first place is _?_ place; the second place is _?_ place; the third place is _?_ place.

Copy the following tables, leaving blanks where there are question marks. Then fill in the blanks to show two ways of writing the same number.

3.

fraction	$\frac{3}{10}$	?	$\frac{1}{10}$	?	$\frac{1}{100}$	?	?	$\frac{7}{100}$
decimal	?	.7	?	1.5	?	.13	3.01	?

4.

fraction	?	$\frac{7}{1000}$	?	$\frac{37}{1000}$	$\frac{231}{1000}$	?	?	$\frac{4}{1000}$
decimal	.001	?	.175	?	?	7.029	1.005	?

5. Complete these sentences.
   One dollar is worth _?_ times as much as a dime.
   One dime is worth _?_ times as much as a cent.
   One cent is worth _?_ times as much as a mill.

6. Write the following decimals in order of their size beginning with the smallest: .05 .75 .16 .09 .99 .01

7. Change the following decimals to common fractions in lowest terms: .5 .1 .40 .75 .250 .600

8. Change the following fractions to decimal hundredths:

   $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{5}$ $\frac{3}{10}$ $\frac{4}{5}$ $\frac{3}{4}$

Subtract. Check by adding.

9.

.9	8.6	$7.00	6.530	8.000
.6	.8	1.26	3.273	4.259

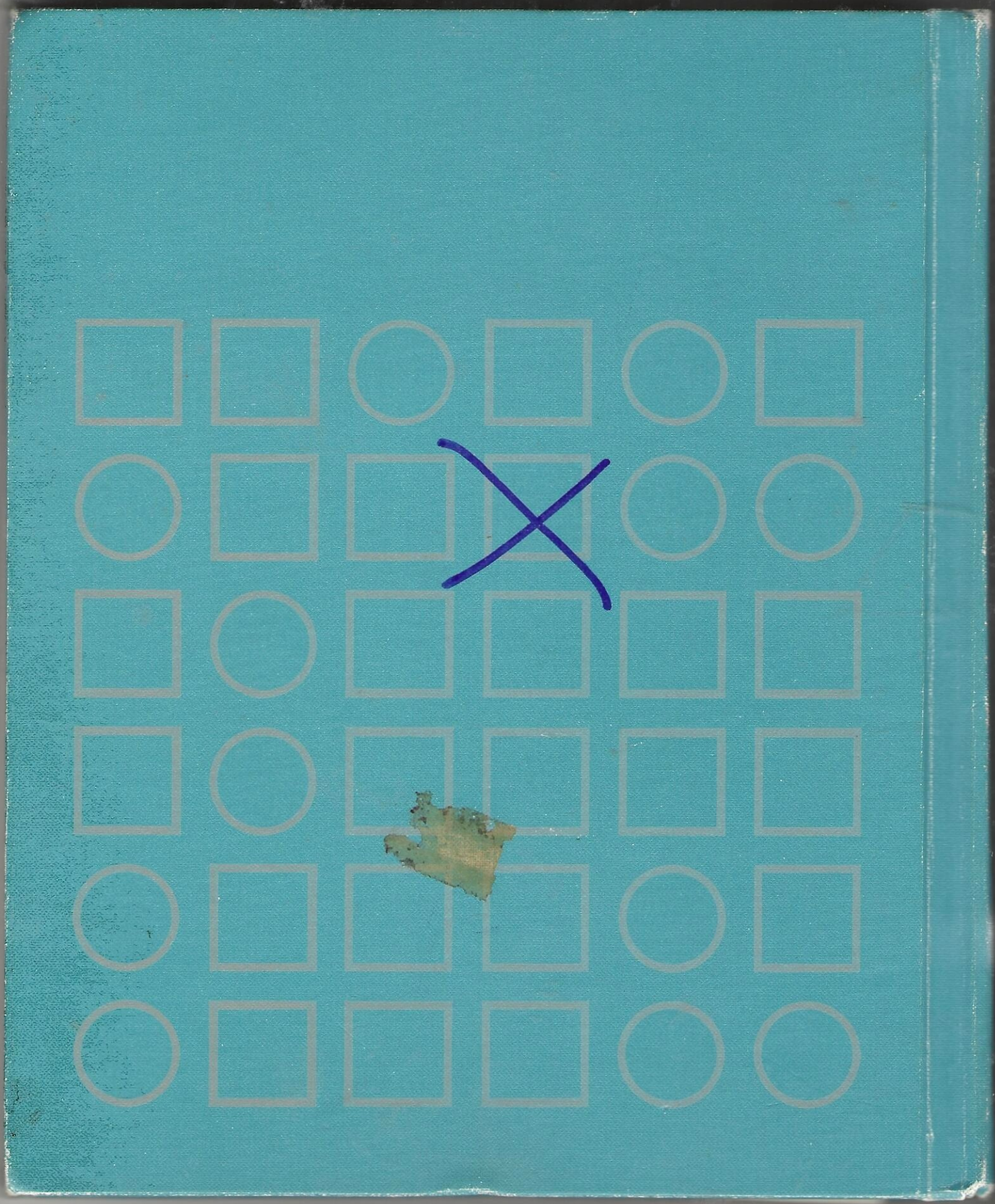
Add down. Check by adding up.

10.

.7	14.2	$4.81	7.211	.237
.8	.8	6.90	.058	.056
.6	5.4	3.00	5.309	.802

What Have You Learned about Decimals?

CHAPTER TEST ON PROBLEM SOLVING

1. The food industry estimates that the average American eats 7.7 pounds of cheese in a year. If Mr. and Mrs. Weber each eat an average amount, how many pounds of cheese will Mr. and Mrs. Weber together eat this year?
2. The average ride on a New York subway takes 17.3 minutes. It takes 25.5 minutes to ride from Sandra's home to Times Square. How much longer is this than an average ride?
3. Sue rode her bicycle .7 of a mile from her home to Jane's house. Then Sue and Jane rode .8 of a mile to the pool. How far did Sue ride to get from her house to the pool?
4. One spring day 2.06 inches of rain fell in San Francisco. The same day 1.49 inches fell in New Orleans. Which city had more rain? How much more?
5. On a day that Atlantic City had .46 of an inch of rain, Galveston had .50 of an inch of rain. How much more rain did Galveston have than Atlantic City?
6. In Glenville 8.16 inches of snow fell on Saturday. On Sunday 5.79 inches of snow fell. How many inches of snow fell in the two days?
7. A winning bobsled team ran the course in 5 minutes 7.85 seconds. The second team made the run in 5 minutes 15.06 seconds. Find the difference in their times.
8. In a certain state gasoline costs 18.5¢ a gallon plus taxes. The state tax is 7.0¢; the federal tax is 3.0¢. Find the price of a gallon of gasoline in that state.

To be continued in Part 2 of this book.